Math Home
Logic is the magic of Mathematics
Practice more & increase your creativity

(differentiation)

1. $\frac{d}{dx}(x^n) = nx^{n-1}$
2. $\frac{d}{dx}(x) = 1$
3. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
4. $\frac{d}{dx}(\text{ constant }) = 0$
5. $\frac{d}{dx}(a^x) = a^x.\ln a, a > 0$
6. $\frac{d}{dx}(e^x) = e^x$
7. $\frac{d}{dx}(e^{mx}) = me^{mx}$
8. $\frac{d}{dx}(\ln\ x) = \frac{1}{x}, x > 0$
9. $\frac{d}{dx}(\sin\ x) = \cos\ x$
10. $\frac{d}{dx}(\cos\ x) = -\sin\ x$
11. $\frac{d}{dx}(\tan\ x) = \sec^2\ x$
12. $\frac{d}{dx}(\cot\ x) = -\mathrm{cosec}^2\ x$
13. $\frac{d}{dx}(\sec\ x) = \sec x\ \tan\ x$
14. $\frac{d}{dx}(\mathrm{cosec}\ x) = -\mathrm{cosec} x\ \cot\ x$
15. $\frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$
16. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}(u) - u\frac{d}{dx}(v)}{v^2}$
17. $\frac{d}{dx}(\sin^{-1}\ x) = \frac{1}{\sqrt{1-x^2}}$
18. $\frac{d}{dx}(\cos^{-1}\ x) = \frac{-1}{\sqrt{1-x^2}}$
19. $\frac{d}{dx}(\tan^{-1}\ x) = \frac{1}{1+x^2}$
20. $\frac{d}{dx}(\cot^{-1}\ x) = \frac{-1}{1+x^2}$
21. $\frac{d}{dx}(\sec^{-1}\ x) = \frac{1}{x\sqrt{x^2-1}}$
22. $\frac{d}{dx}(\mathrm{cosec}^{-1}\ x) = \frac{-1}{x\sqrt{x^2-1}}$

(Integration)

1 . $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

2 . $\int dx = x + c$

3 . $\int \frac{1}{2\sqrt{x}} dx = \sqrt{x} + c$

4 . $\int 0 dx = c$

5 . $\int a^x dx = \frac{a^x}{\ln\ a} + c$

6. $\int e^x dx = e^x + c$

7 . $\int e^{mx} dx = \frac{1}{m} e^{mx} + c$

8 . $\int \frac{1}{x} dx = \ln\ |x| + c$

9 . $\int \sin x dx = -\cos x + c$

10 . $\int \cos\ x\ dx = \sin\ x + c$

11 . $\int \tan\ x dx = \ln\ |\sec\ x| + c$

12 . $\int \sec x\ dx = \ln\ |\sec\ x + \tan\ x| + c$
$= \ln\ \left|\tan\ \left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$

13 . $\int \mathrm{cosec}\ x dx = -\ln\ |\mathrm{cosec}\ x + \cot x\ | + c$
$= \ln\ \left|\tan\ \frac{x}{2}\right| + c$

14. $\int \sec^2\ x\ dx = \tan\ x + c$

15. $\int \mathrm{cosec}^2 x\ dx = -\cot x + c$

16 . $\int \sec x \tan x\ dx = \sec\ x + c$

17 . $\int \mathrm{cosec} x\ \cot\ x dx = -\mathrm{cosec}\ x + c$

18 . $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$ (ii) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

19 . $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\ \frac{x}{a}$ +c (ii) $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$

20 . $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}\ x + c$

21 . $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\ \left|\frac{x-a}{x+a}\right| + c$

22 . $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\ \left|\frac{a+x}{a-x}\right| + c$

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23 . $\int \frac{f'(x)}{\sqrt{f(x)}} \mathrm{dx} = 2\sqrt{f(\mathrm{x})} + \mathrm{c}$

24. $\int \frac{f'(x)}{f(x)} \mathrm{d}x = \ln \mid f(x) \mid +c$

25. $\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$

26. $\int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2} \ln \left| x + \sqrt{x^2 - a^2} \right|$

27. $\int \sqrt{x^2 + a^2} dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2} \ln \mid x + \sqrt{x^2 + a^2}$

28. $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln \left| x + \sqrt{x^2 + a^2} \right| + c$

29. $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right| + \mathrm{c}$

30. $\int \mathrm{uv}\, dx = u \int v dx - \int \left\{ \frac{d}{dx}(u) \int v dx \right\} dx$

UV মেথডে U এবং V কে

"*LIATE*" এর মাধ্যমে মনে রাখা যায় ।

এ শব্দের ক্রমে যে বর্ণ প্রথমে থাকবে তাকে প্রথম ফাংশন ধরতে হবে ।

এখানে,

L = Logarithmic,

I = Inverse Circular,

A = Algebraic,

T = Trigonometric

E = Exponential

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Exercise- 10.1

1)Integrate: $\int x^3 dx$

Solution: $\int x^3 dx = \frac{x^{3+1}}{3+1} + c = \frac{1}{4}x^4 + c$

1(ii) Integrate: $\int 5x^7 dx$

Solution: $\int 5x^7 dx = 5\int x^7 dx = 5 \cdot \frac{x^{7+1}}{7+1} + c$

$$= \frac{5}{8}x^8 + c$$

1(iii) Integrate: $\int dx$

Solution: $\int dx = \int x^0 dx = \frac{x^{0+1}}{0+1} + c = \frac{x}{1} + c$

$$= x + c$$

1(iv) Integrate: $\int \mathbf{dt}$

Solution: $\int \mathrm{dt} = \mathrm{t} + \mathrm{c}$

1(v) Integrate: $\int \frac{dx}{9}$

Solution: $\int \frac{dx}{9} = \frac{1}{9}\int dx = \frac{1}{9}x + c$

1(vi) Integrate: $\int (x^3 + x)dx$

Solution: $\int (x^3 + x)dx = \int x^3 dx + \int x dx$

$$= \frac{1}{4}x^4 + \frac{1}{2}x^2 + c$$

1(vii) Integrate: $\int \frac{y^3 - 8}{y - 2} dy$

Solution: $\int \frac{y^3 - 8}{y - 2} dy - \int \frac{(y-2)(y^2+2y+4)}{(y-2)} d$

$$= \int (y^2 + 2y + 4)dy = \frac{1}{3}y^3 + y^2 + 4y + c$$

2(i) Integrate: $\int \frac{1}{3\sqrt{x}} dx$

Solution: $\int \frac{1}{3\sqrt{x}} dx = \frac{2}{3}\int \frac{1}{2\sqrt{x}} dx = \frac{2}{3}\sqrt{x} + c$

2(ii) Integrate: $\int \sqrt[3]{x} dx$

Solution: $\int \sqrt[3]{x} dx = \int x^{\frac{1}{3}} dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c = \frac{3}{4}x^{\frac{3}{4}}.$

2(iii) Integrate: $\int \left(1 + \frac{1}{3}x^2 - \frac{1}{2\sqrt{x}}\right) dx$

Solution

$$= \int 1\, dx + \frac{1}{3}\int x^2 dx - \int \frac{1}{2\sqrt{x}} dx$$

$= X + \frac{1}{9}x^3 - \sqrt{x} + C$

2(iv) Integrate: $\int \left(x^2 + \frac{1}{x^2}\right)^2 dx$

Solution: $\int \left(x^2 + \frac{1}{x^2}\right)^2 dx = \int \left(x^4 + 2 + \frac{1}{x^4}\right)$ d

$= \int x^4 dx + 2\int dx + \int x^{-4} dx$

$$= \frac{1}{5}x^5 + 2x - \frac{1}{3}x^{-3} + c = \frac{1}{5}x^3 + 2x - \frac{1}{3x^3} + c$$

3(i) Integrate: $\int \frac{\sin x dx}{\cos^2 x}$

$= \int \sec x \tan x\, dx = \sec x + c$

3(ii) Integrate: $\int \sec x(\sec x - \tan x)\, dx$

[BB. 16]

Solution: $\int \sec x(\sec x - \tan x)\, dx$

$$= \int \sec^2 x\, dx - \int \sec x \tan x dx$$

$$= \tan x - \sec x + c$$

3(iii)Integrate: $\int \sqrt{1 - \cos 2x} dx$

[CtgB, 05, 09,12; BB. 08; SB. 06]

Solution: $\int \sqrt{1 - \cos 2x} dx = \int \sqrt{2\sin^2 x} dx$

$= \sqrt{2}\int \sin x dx = -\sqrt{2}\cos x + c$

3(iv) Integrate: $\int \frac{1}{1 + \cos 2x} dx$ **[CB. 08]**

Solution: $\int \frac{1}{1+\cos 2x} dx = \int \frac{1}{2\cos^2 x} dx$

$$\frac{1}{2}\int \sec^2 x\, dx = \frac{1}{2}\tan x + c$$

4) $\int \sec^2 x \operatorname{cosec}^2 x dx$ **[DB. 12, 07, 09; CB. 05, 11: DjB. 11:Raj. 08, 10: SR. 04, 10, 14; BB. 09,04 :CigB. 03, 08, 14; MB. 05; JB. 07]**

Solution: $\int \sec^2 x \operatorname{cosec}^2 x\, dx$

$$= \int \frac{1}{\cos^2 x} \frac{1}{\sin^2 x} dx$$

$$= \int \frac{1}{\cos^2 x \sin^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx$$

$$= \int \left(\frac{\sin^2 x}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\cos^2 x \sin^2 x}\right) dx$$

$$= \int (\sec^2 x + \operatorname{cosec}^2 x)dx = \tan x - \cot x + c$$

Exercise-10.2

1(i) Integrate: $\int (5x+3)^6 dx$

Solution: $\int (5x+3)^6 dx = \frac{(5x+3)^{6+1}}{5(6+1)} + c$

$= \frac{1}{35}(5x+3)^7 + c$

1(ii) Integrate: $\int (1-x)^7 dx$

Solution: $\int (1-x)^7 dx = \frac{(1-x)^{7+1}}{(-1)\cdot(7+1)} + c$

$= -\frac{1}{8}(1-x)^8 + c$

1(iii) Integrate: $\int \sqrt{2x+3}dx$

Solation: $\int \sqrt{2x+3}dx = \int (2x+3)^{\frac{1}{2}}dx$

$= \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}\cdot 2} + c = \frac{1}{3}(2x+3)^{\frac{3}{2}} + c$

2(i) Integrate: $\int \frac{dx}{\sqrt{x}-\sqrt{x-1}}$

Solution: $\int \frac{dx}{\sqrt{x}-\sqrt{x-1}}$

$= \int \frac{\sqrt{x}+\sqrt{x-1}}{(\sqrt{x}-\sqrt{x-1})(\sqrt{x}+\sqrt{x-1})}dx$

$= \int \frac{\sqrt{x}+\sqrt{x-1}}{x-(x-1)}dx = \int (\sqrt{x}+\sqrt{x-1})dx$

$= \int x^{\frac{1}{2}}dx + \int (x-1)^{\frac{1}{2}}dx$

$= \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + c$

$= \frac{2}{3}\left\{x^{\frac{3}{2}} + (x-1)^{\frac{2}{3}}\right\} + c$

2(ii) Integrate: $\int \frac{dx}{\sqrt{x+1}+\sqrt{x-1}}$ **[DjB. 10; RB. 02]**

Solution: $\int \frac{dx}{\sqrt{x+1}+\sqrt{x-1}}$

$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{(\sqrt{x+1}+\sqrt{x-1})(\sqrt{x+1}-\sqrt{x-1})}dx$

$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{(x+1)-(x-1)}dx$

$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{2}dx$

$= \frac{1}{2}\int (x+1)^{\frac{1}{2}}dx - \frac{1}{2}\int (x-1)^{\frac{1}{2}}dx$

$= \frac{1}{2}\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{1}{2}\cdot\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{1}{2}\cdot\frac{2}{3}(x-1)^{\frac{3}{2}} + c$

$= \frac{1}{3}\left\{(x+1)^{\frac{3}{2}} - (x-1)^{\frac{3}{2}}\right\} + c$

2(iii) Integrate: $\int \cos(\alpha - 5x)dx$

Solution: $\int \cos(\alpha-5x)dx$

$= \frac{\sin(\alpha-5x)}{-5} + c = -\frac{1}{5}\sin(\alpha-5x) + c$

3(i) Integrate: $\int \sin x^\circ \mathrm{dx}$ **[CB. 02; CtgB. 04]**

Solution: $\int \sin x^0 dx = \int \sin\frac{\pi x}{180}dx$

$= \frac{-\cos\frac{\pi x}{180}}{\frac{\pi}{180}} + c = -\frac{180}{\pi}\cdot\cos\frac{\pi x}{180} + c$

3(ii) Integrate: $\int \sqrt{1+\cos x}\,dx$

Solution: $\int \sqrt{1+\cos x}\,dx = \int \sqrt{2\cos^2\frac{x}{2}}dx$

$= \sqrt{2}\int \cos\frac{x}{2}dx = \sqrt{2}\cdot\frac{\sin\frac{x}{2}}{\frac{1}{2}} + c$

$= 2\sqrt{2}\sin\frac{x}{2} + c$

3(iii) Iategrate: $\int \cot^2\frac{x}{2}dx$

Solution: $\int \cot^2\frac{x}{2}dx = \int \left(\mathrm{cosec}^2\frac{x}{2} - 1\right)dx$

$= -\frac{\cot\frac{x}{2}}{\frac{1}{2}} - x + c = -2\cot\frac{x}{2} - x + c$

4(i) Integrate: $\int \sin 5x \sin 3x\, dx$ **[Ctg $B.12$;**

BB. 08, 12: JB.12]

Solution: $\int \sin 5x \sin 3x \, dx$

$= \frac{1}{2}\int 2\sin 5x \,.\sin 3x \, dx$

$= \frac{1}{2}\int (\cos 2x - \cos 8x)dx$

$= \frac{1}{2}\left(\frac{\sin 2x}{2} - \frac{\sin 8x}{8}\right) + c$

$= \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + c$

4(ii) Integrate: $\int$ 7sin4x sin 2x dx

[DB. 11; RB, 05;]

Solution: $\int 7\sin 4x \cdot \sin 2x \, dx$

$= \frac{7}{2}\int 2\sin 4x \cdot \sin 2x \, dx$

$= \frac{7}{2}\int (\cos 2x - \cos 6x)dx$

$= \frac{7}{2}\left(\frac{\sin 2x}{2} - \frac{\sin 6x}{6}\right) + c$

$= \frac{7}{12}(3\sin 2x - \sin 6x) + c$

4(iii) Integrate: $\int 3\cos 3x \cdot \cos x \, dx$

Solution: $\int 3\cos 3x \cdot \cos x \, dx$

$= \frac{3}{2}\int 2\cos 3x \cdot \cos x \, dx$

$= \frac{3}{2}\int (\cos 4x + \cos 2x)dx$

$= \frac{3}{2}\left(\frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right) + c$

$= \frac{3}{4}\left(\sin 2x + \frac{1}{2}\sin 4x\right) + c$

4(iv) Integrate: $\int 5\cos 4x \sin 3x dx$

[DjB. 14; SB, 14; BR, 12, 08; CB12]

Solution: $\int 5\cos 4x \sin 3x \, dx$

$= \frac{5}{2}\int 2\cos 4x \cdot \sin 3x dx$

$= \frac{5}{2}\int (\sin 7x - \sin x)dx$

$= \frac{5}{2}\left\{\frac{-\cos 7x}{7} - (-\cos x)\right\} + c$

$= \frac{5}{2}\left(\cos x - \frac{1}{7}\cos 7x\right) + c$

5(i)integrate: $\int \sin^2 3\theta \, d\theta$

Solation: $\int \sin^2 3\theta d\theta = \frac{1}{2}\int 2\sin^2 3\theta \, d\theta$

$= \frac{1}{2}\int (1 - \cos 6\theta)d\theta = \frac{1}{2}\left(\theta - \frac{\sin 6\theta}{6}\right) + c$

5(ii) Integrate: $\int \left(1 + \cos^2 \frac{x}{2}\right) dx$ [BUTEX, 06 − 07]

Solution: $\int \left(1 + \cos^2 \frac{x}{2}\right) dx$

$= \int 1 dx + \int \cos^2 \frac{x}{2} dx$

$= x + \frac{1}{2}\int 2\cos^2 \frac{x}{2} dx = x + \frac{1}{2}\int (1 + \cos x)dx$

$= x + \frac{1}{2}(x + \sin x) + c$

$= x + \frac{1}{2}x + \frac{1}{2}\sin x + c$

$= \frac{3}{2}x + \frac{1}{2}\sin x + c$

5(iii) Integrate: $\int \sin^3 x dx$

Solution: $\int \sin^3 x dx = \frac{1}{4}\int 4\sin^3 x dx$

$= \frac{1}{4}\int (3\sin x - \sin 3x)dx$

$= \frac{1}{4}\left\{(-3\cos x) + \frac{1}{3}\cos 3x\right\} + c$

$= \frac{1}{12}\cos 3x - \frac{3}{4}\cos x + c$

5(iv) Integrate: $\int \cos^3 x dx$

Solution: $\int \cos^3 x dx = \frac{1}{4}\int 4\cos^3 x dx$

$= \frac{1}{4}\int (3\cos x + \cos 3x)dx$

$= \frac{1}{4}\left(3\sin x + \frac{\sin 3x}{3}\right) + c$

$= \frac{1}{12}(9\sin x + \sin 3x) + c$

6) Integrate: $\int \sin^4 xdx$

Solution: $\int \sin^4 xdx = \int (\sin^2 x)^2 dx$

$= \int \frac{1}{4}(2\sin^2 x)^2 dx$

$= \int \frac{1}{4}(1-\cos 2x)^2 dx$

$= \frac{1}{4}\int (1-2\cos 2x+\cos^2 2x)dx$

$= \frac{1}{4}\int dx - \frac{2}{4}\int \cos 2xdx + \frac{1}{4}\int \cos^2 2xdx$

$= \frac{1}{4}\int dx - \frac{2}{4}\int \cos 2xdx + \frac{1}{4.2}\int 2\cos^2 2xdx$

$= \frac{x}{4} - \frac{1}{2}\cdot\frac{\sin 2x}{3} + \frac{1}{8}\int (1+\cos 4x)dx$

$= \frac{x}{4} - \frac{1}{4}\sin 2x + \frac{1}{8}\left(x + \frac{\sin 4x}{4}\right) + c$

$= \frac{x}{4} + \frac{x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c$

$= \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c$

$= \frac{1}{4}\left(\frac{3x}{2} - \sin 2x + \frac{1}{8}\sin 4x\right) + c$

7(i) Integrate: $\int (2\cos x + \sin x)\cos xdx$

Solution: $\int (2\cos x + \sin x)\cos xdx$

$= \int 2\cos^2 xdx + \int \sin x.\cos xdx$

$= \int (1+\cos 2x)dx + \frac{1}{2}\int \sin 2xdx$

$= \left(x + \frac{\sin 2x}{2}\right) + \frac{1}{2}\left(-\frac{\cos 2x}{2}\right) + c$

$= x + \frac{\sin 2x}{2} - \frac{1}{4}\cos 2x + c$

7(ii) Integrate: $\int \sin^2 x\cos 2xdx$

[RR. 12; SB. 11; CB. 10, 13, 14; CtgB. 09; JB, 05; RB. 13]

Solution: $\int \sin^2 x\cdot\cos 2x\,dx$

$= \frac{1}{2}\int 2\sin^2 x\cdot\cos 2x\,dx$

$= \frac{1}{2}\int (1-\cos 2x)\cos 2x\,dx$

$= \frac{1}{2}\int \cos 2xdx - \frac{1}{2}\int \cos^2 2xdx$

$= \frac{1}{2}\int \cos 2x\,dx - \frac{1}{2.2}\int 2\cos^2 2x\,dx$

$= \frac{1}{2}\frac{\sin 2x}{2} - \frac{1}{4}\int (1+\cos 4x)dx$

$= \frac{1}{4}\sin 2x - \frac{1}{4}\left(x + \frac{\sin 4x}{4}\right) + c$

$= \frac{1}{4}\left(\sin 2x - x - \frac{1}{4}\sin 4x\right) + c$

7(iii) Integrate: $\int \sin^2 x\cos^2 x\,dx$

[DB. 13; JB. 08; CtgB. 06]

Solution: $\int \sin^2 x\cdot\cos^2 x\,dx$

$= \frac{1}{4}\int (2\sin x\cdot\cos x)^2 dx = \frac{1}{4}\int \sin^2 2x\,dx$

$= \frac{1}{8}\int 2\sin^2 2x\,dx = \frac{1}{8}\int (1-\cos 4x)dx$

$= \frac{1}{8}x - \frac{1}{8}\frac{\sin 4x}{4} + c = \frac{1}{32}(4x - \sin 4x) + c$

7(iv) Integrate: $\int \sin^3 x\cos^3 x\,dx$

Solution: $\int \sin^3 x\cdot\cos^3 xdx$

$= \int \frac{1}{8}(2\sin x\cos x)^3 d$

$= \frac{1}{8}\int \sin^3 2x\,dx = \frac{1}{8}\cdot\frac{1}{4}\int 4\sin^3 2x\,dx$

$= \frac{1}{32}\int (3\sin 2x - \sin 6x)dx$

$= \frac{1}{32}\left(\frac{-3\cos 2x}{2} + \frac{\cos 6x}{6}\right) + c$

7(v) Integrate: $\int \sin^4 x\cos^4 xdx$

Solution:

$\int \sin^4 x\cdot\cos^4 x\,dx$

$= \frac{1}{16}\int (2\sin x\,.\cos x)^4 dx$

$= \frac{1}{16}\int (\sin 2x)^4 dx = \frac{1}{16\times 4}\int (2\sin^2 2x)^2 dx$

$= \frac{1}{64}\int (1-\cos 4x)^2 dx$

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$$= \frac{1}{64}\int (1-2\cos\ 4x+\cos^2\ 4x)dx$$
$$= \frac{1}{64}\int 1\text{dx} - \frac{1}{32}\int \cos\ 4x\text{dx} + \frac{1}{64}\int \cos^2\ 4x\ dx$$
$$= \frac{1}{64}x - \frac{1}{32}\cdot\frac{\sin\ 4x}{4} + \frac{1}{128}\int 2\cos^2\ 4x\ dx$$
$$= \frac{1}{64}x - \frac{\sin\ 4x}{128} + \frac{1}{128}\int (1+\cos\ 8x)dx$$
$$= \frac{1}{64}x - \frac{\sin\ 4x}{128} + \frac{1}{128}\left(x+\frac{\sin\ 8x}{8}\right) + c$$
$$= \frac{1}{128}\left(3x - \sin\ 4x + \frac{\sin\ 8x}{8}\right) + c$$

8(i) Integrate: $\int \frac{1}{1+\cos\ x}dx$ **[JB. 11; CB. 13, 05]**

Solution: $\int \frac{1}{1+\cos\ x}dx = \int \frac{1}{2\cos^2\frac{x}{2}}dx$

$$= \frac{1}{2}\int \sec^2\frac{x}{2}\ dx = \frac{1}{2}\frac{\tan\ \frac{x}{2}}{\frac{1}{2}} + c = \tan\ \frac{x}{2} + c$$

Alternative method: $\int \frac{1}{1+\cos\ x}dx$

$$= \int \frac{(1-\cos\ x)}{(1+\cos\ x)(1-\cos\ x)}dx$$
$$= \int \frac{(1-\cos\ x)}{1-\cos^2\ x}dx = \int \frac{(1-\cos\ x)}{\sin^2\ x}dx$$
$$= \int \left(\frac{1}{\sin^2\ x} - \frac{\cos\ x}{\sin\ x\cdot\sin\ x}\right)dx$$
$$= \int \text{cosec}^2\ xdx - \int \text{cosec}\ x\cdot\cot\ xdx$$
$$= -\cot\ x + \text{cosec}\ x + c = \text{cosec}\ x - \cot\ x + c$$

$$= \frac{1}{\sin\ x} - \frac{\cos\ x}{\sin\ x} + c = \frac{1-\cos\ x}{\sin\ x} + c$$
$$= \frac{2\sin^2\ \frac{x}{2}}{2\sin\frac{x}{2}\cos\ \frac{x}{2}} + c = \tan\ \frac{x}{2} + c$$

8(ii) Integrate: $\int \frac{1}{1+\sin\ x}dx$ **[.JB. 13; BUET. 03 – 04]**

Solution: $\int \frac{1}{1+\sin\ x}dx = \int \frac{(1-\sin\ x)}{(1+\sin\ x)(1-\sin\ x)}dx$

$$= \int \frac{(1-\sin\ x)}{1-\sin^2\ x}dx = \int \frac{(1-\sin\ x)}{\cos^2\ x}dx$$
$$= \int \left(\frac{1}{\cos^2\ x} - \frac{\sin\ x}{\cos\ x\cdot\cos\ x}\right)dx$$
$$= \int \sec^2 x\ dx - \int \text{secx}\cdot\tan\ xdx$$
$$= \tan\ x - \sec\ x + c$$

8(iii) Integrate: $\int \frac{1}{1-\sin\ x}dx$ **[07; SB. 13]**

Solution: $\int \frac{1}{1-\sin\ x}dx = \int \frac{(1+\sin\ x)}{(1+\sin\ x)(1-\sin\ x)}dx$

$$= \int \frac{(1+\sin\ x)}{1-\sin^2\ x}dx = \int \frac{1+\sin\ x}{\cos^2\ x}dx$$
$$= \int \left(\frac{1}{\cos^2\ x} + \frac{\sin\ x}{\cos\ x\cdot\cos\ x}\right)dx$$
$$= \int \sec^2\ xdx + \int \sec x\,.\tan x\ dx$$
$$= \tan\ x + \sec\ x + c$$

(9) Integrate:

$\int \frac{e^{5x}+e^{3x}}{e^x+e^{-x}}dx$; $[DjB.10; CtgB.00]$

Solution: $\int \frac{e^{5x}+e^{3x}}{e^x+e^{-x}}dx \quad = \int \frac{e^{4x}(e^x+e^{-x})}{(e^x+e^{-x})}dx$

$$= \int e^{4x}dx = \frac{1}{4}e^{4x} + c$$

10(i) Integrate: $\int \mathbf{a^{4x}dx}$

Solution: $\int a^{4x}dx = \frac{a^{4x}}{\ln\ a\cdot 4} + c = \frac{a^{4x}}{4\ln\ a} + c$

10(ii) Integrate: $\int \left(\frac{3}{x-1} - \frac{4}{x-2}\right)dx$

Solution: $\int \left(\frac{3}{x-1} - \frac{4}{x-2}\right)dx$

$$= 3\ln|x-1| - 4\ln|x-2| + c$$

10(iii) Integrate: $\int \frac{xdx}{(x-1)}$ $[SB.17]$

Solution: $\int \frac{xdx}{(x-1)}$

$$= \int \frac{x-1+1}{x-1}dx = \int \frac{x-1}{x-1}dx + \int \frac{1}{x-1}dx$$
$$= \int 1dx + \int \frac{1}{x-1}dx = x + \ln\ |x-1| + c$$

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Exercise-10.3

1(i) Integrate: $\int xe^{x^2}dx$

Solution: $\therefore \int xe^{x^2}dx$

ধরি, $x^2 = z$
$\Rightarrow 2xdx = dz$
$\therefore xdx = \frac{1}{2}dz$

$= \frac{1}{2}\int e^z dz$

$= \frac{1}{2}e^z + c$

$= \frac{1}{2}e^{x^2} + c$

1(ii) Integrate:

$\int x^7 e^{x^8} dx$

Solution: $\therefore \int x^7 e^{x^8} dx$

ধরি, $x^8 = z$
$\Rightarrow 8x^7dx = dz$
$\therefore x^7dx = \frac{1}{8}dz$

$= \frac{1}{8}\int e^z dz$

$= \frac{1}{8}e^z + c$

$= \frac{1}{8}e^{x^8} + c$

1(iii) Integrate: $\int \cos x\, e^{\sin x}dx$; **[DB.11; RB. 04]**

Solution: $\therefore \int \cos x \cdot e^{\sin x} dx$

ধরি, $\sin x = z$
$\Rightarrow \cos x\, dx = dz$

$= \int e^z dz$

$= e^z + c$

$= e^{\sin x} + c$

1(iv) Integrate: $\int \sec^2 x\, e^{\tan x} dx$

Solution: $\therefore \int \sec^2 x \cdot e^{\tan x}\, dx$

ধরি, tan x = z

$\sec^2 x\, dx = dz$

$= \int e^z dz$

$= e^z + c$

$= e^{\tan x}$

1(v) Integrate: $\int \frac{e^{\sqrt{x}}}{5\sqrt{x}}dx$

Solution: $\int \frac{e^{\sqrt{x}}}{5\sqrt{x}}dx$

ধরি, $\sqrt{x} = z$

$\therefore \frac{1}{2\sqrt{x}}dx = dz$

$= \frac{2}{5}\int e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}dx$

$= \frac{2}{5}\int e^z dz$

$= \frac{2}{5}e^z + c = \frac{2}{5}e^{\sqrt{x}} + c$

1(vi) Integrate: $\int \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}}dx$

Solution: $\therefore \int \left(1 - \frac{1}{x^2}\right) e^{x+\frac{1}{x}}dx$

ধরি,, $x + \frac{1}{x} = z$
$\therefore \left(1 - \frac{1}{x^2}\right)dx = dz$

$= \int e^z dz$

$= e^z + c$

$= e^{x+\frac{1}{x}} + c$

1(vii) Integrate: $\int x\, a^{x^2}dx$

Solution: $\therefore$

$\int xa^{x^2}dx$

ধরি, $x^2 = z$
$\Rightarrow 2xdx = dz$
$\therefore xdx = \frac{1}{2}$

$= \frac{1}{2}\int a^z dz$

$= \frac{1}{2}\frac{a^z}{\ln(a)} + c$

$= \frac{1}{2}\frac{a^{x^2}}{\ln(a)} + c$

2(i) Integrate: $\int \sin^3 x \cos^3 x dx$

Solution:: $\int \sin^3 x \cdot \cos^3 x\, dx$

$= \int \sin^3 x \cdot \cos^2 x \cdot \cos xdx$

$= \int \sin^3 x(1 - \sin^2 x)\cos xdx$

ধরি, $\sin x = z$

$\therefore \cos xdx = dz$

$= \int z^3(1 - z^2)dz = \int (z^3 - z^5)dz$

$= \frac{1}{4}z^4 - \frac{1}{6}z^6 + c = \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + c$

2(ii) Integrate: $\int \sin^5 x\, dx$ **[RUET. 10-11]**

Solution: $\int \sin^5 x\, dx$

ধরি, $\cos x = z$
$-\sin x\, dx = dz$

$= \int \sin^4 x \cdot \sin x\, dx$

$= \int (\sin^2 x)^2 \cdot \sin xdx$

$= \int (1 - \cos^2 x)^2 \cdot \sin xdx$

$= \int (1 - z^2)^2(-dz)$

$= \int -(1 - 2z^2 + z^4)dz$

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$$= -\left(z - \frac{2}{3}z^3 + \frac{1}{5}z^2\right) + c$$

$$= -\left(\cos\ x - \frac{2}{3}\cos^3\ x + \frac{1}{5}\cos^2\ x\right) + c$$

2(iii) Integrate: $\int \frac{1+\cos x}{x+\sin x} dx$; **[JB. 04]**

Solution: $\therefore \int \frac{1+\cos x}{x+\sin x} dx$

ধরি, $x + \sin x = z$ $\Rightarrow (1 + \cos x)dx = dz$

$= \int \frac{dz}{z}$

$= \ln|z| + c$

$= \ln|x + \sin x| + c$

2(iv) Integrate:

$\int \frac{\sin x}{1+\cos x} dx$

ধরি, $1 + \cos x = z$ $\Rightarrow -\sin x dx = dz$ $\Rightarrow \sin\ x dx = -dz$

Solution: $\therefore \int \frac{\sin\ x}{1+\cos\ x} dx$

$= -\int \frac{dz}{z} = -\ln\ |z| + c$

$= -\ln\ |1 + \cos\ x| + c$

2(v) Integrate: $\int \frac{\sin\ x}{3+4\cos\ x} dx$ **[DR. 15; BB. 13]**

Solution: $= \int \frac{\sin x}{3+4\cos x} dx$

ধরি, $3 + 4\cos x = z$ $\Rightarrow 0 - 4\sin x dx = dz$ $\therefore \sin x dx = -\frac{1}{4} dx$

$= -\int \frac{1}{4}\frac{dz}{z} = -\frac{1}{4}\ln|z| + c$

$= -\frac{1}{4}\ln\ |3 + 4\cos\ x| + c$

3(i) Integrate: $\int \frac{1-\tan\ x}{1+\tan\ x} dx$

Solution: $\int \frac{1-\tan\ x}{1+\tan\ x} dx = \int \frac{1-\frac{\sin\ x}{\cos\ x}}{1+\frac{\sin\ x}{\cos\ x}} dx$

$= \int \frac{\cos\ x - \sin\ x}{\cos\ x + \sin\ x} dx = \ln\ |\cos\ x + \sin\ x| + c$

[Formula: $\int \frac{f'(x)}{f(x)} dx = \ln\ f(x)\ | + c$]

3(ii) Integrate: $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$ **[DjB.10]**

Solution: ধরি,, $e^x + e^{-x} = z$

$\Rightarrow (e^x - e^{-x})dx = dz$

$\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \int \frac{1}{z} dz = \ln\ |z| + c$

$= \ln|e^x + e^{-x}| + c$

[Fomula: $\int \frac{f'(x)}{f(x)} dx = \ln\ |f(x)| + c$]

3(iii) Integrate: $\int \frac{1}{e^x+1} dx$ **: [JB. 10]**

Solution: $\int \frac{1}{e^x+1} dx$

$= \int \frac{e^{-x}}{1 + e^{-x}} dx$

[e^{-x} দ্বারা লব ও হরকে গুন করি]

ধরি, $1 + e^{-x} = z$

$\therefore e^{-x} dx = -dz$

$= -\int \frac{dz}{z} = -\ln|z| + c$

$= -\ln|1 + e^{-x}| + c$

4(i) Integrate: $\int \frac{\cos\ x}{\sqrt{\sin\ x}} dx$; **[RB.10; CB. 05]**

Solution: $\therefore \int \frac{\cos\ x}{\sqrt{\sin\ x}} dx$

ধরি,, $\sin x = z$ $\therefore \cos\ x dx = dz$

$= \int \frac{1}{\sqrt{z}} dz$

$= 2\int \frac{1}{2\sqrt{z}} dz$

$= 2\sqrt{z} + c$

$= 2\sqrt{\sin\ x} + c$

4(ii) Integrate: $\int \frac{\sec^2 x\, dx}{\sqrt{1+\tan x}}$ **[DB.00]**

Solution: $\therefore \int \frac{\sec^2 x dx}{\sqrt{1+\tan x}}$

ধরি, $1 + \tan x = z$ $\therefore \sec^2 x dx = dz$

$= \int \frac{dz}{\sqrt{z}}$

$= 2\int \frac{1}{2\sqrt{z}} dz$

$= 2\sqrt{z} + c$

$= 2\sqrt{1+\tan x} + c$

4(iii) Integrate: $\int \frac{x}{\sqrt{1-x^2}} dx$; **DjB.11; JB. 06]**

Solution: $\int \frac{x}{\sqrt{1-x^2}} dx$

$= -\frac{1}{2}\int \frac{dz}{\sqrt{z}}$

$= -\int \frac{1}{2\sqrt{z}} dz$

$= -\sqrt{z} + c$

ধরি, $1 - x^2 = z$
$\Rightarrow -2xdx = dz$
$\therefore xdx = -\frac{1}{2}\mathrm{dz}$

$= -\sqrt{1-x^2} + c$

4(iv) Integrate: $\int \frac{x^3 dx}{\sqrt{1-2x^4}}$ **[Ctg**

Solution: $\therefore \int \frac{x^3 dx}{\sqrt{1-2x^4}}$

$= -\int \frac{1}{8}\frac{dz}{\sqrt{z}}$

$= -\frac{1}{4}\int \frac{1}{2\sqrt{z}} dz$

$= -\frac{1}{4}\sqrt{z} + c$

ধরি, $1 - 2x^4 = z$
$\Rightarrow -8x^3 dx = dz$
$\therefore x^3 dx = -\frac{1}{8}dz$

$= -\frac{1}{4}\sqrt{1-2x^4} + \mathrm{C}$

4(v) Integrate: $\int \frac{\mathrm{dx}}{\mathrm{x}\sqrt{1+\ln \mathrm{x}}}$ **[CB. 03]**

Solution: $\int \frac{dx}{x\sqrt{1+\ln x}}$

$= \int \frac{\frac{1}{x}\cdot dx}{\sqrt{1+\ln x}}$

$= 2\sqrt{1+\ln x} + c$

[Formula: $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$]

4(vi) Integrate: $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$; **[BB. 05]**

solution: $\int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} dx = \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x}\ \cos^2 x} \cdot dx$

$= \int \frac{\sqrt{\tan x}}{\tan x \cdot \cos^2 x} dx$

$= \int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

ধরি, $\tan x = \mathrm{z}$
$\therefore \sec^2 x dx = dz$

$= \int \frac{dz}{\sqrt{\mathrm{z}}} = 2\sqrt{z} + c$

$= 2\sqrt{\tan x} + c$

5(i) Integrate: $\int \frac{\ln x}{x} dx$

Solution: ধরি, $\ln x = z$ $\quad \therefore \frac{1}{x}dx = dz$

$\int \frac{\ln x}{x} dx = \int z dz = \frac{1}{2}z^2 + c = \frac{1}{2}(\ln x)^2 + c$

5(ii) Integrate: $\int \frac{1}{x(1+\ln x)} dx$ **[CB. 12; BB, 09; DB.14]**

Solution: ধরি, $1 + \ln x = z \Rightarrow \frac{1}{x}dx = dz$

$\therefore \int \frac{1}{x(1+\ln x)} dx = \int \frac{dz}{z}$

$= \ln|z| + c = \ln|1+\ln x| + c$

5(iii) Integrate: $\int \frac{\tan x}{\ln(\cos x)} dx$ **|DjB. 14;RB. 09;]**

Solution: ধরি, $\ln(\cos x) = z$

$\Rightarrow \frac{1}{\cos x}(-\sin x)dx = dz. \quad \therefore \tan x dx = -dz$

$\int \frac{\tan x}{\ln(\cos x)} dx = -\int \frac{dz}{z} = -\ln|z| + c$

$= -\ln|\ln(\cos x)| + c$

6(i) Integrate: $\int \frac{\tan^{-1} x}{1+x^2} dx$

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Solution: ধরি, $\tan^{-1} x = z \Rightarrow \frac{1}{1+x^2}dx = dz$

ধরি, $\tan^{-1} x = z$

$\Rightarrow \frac{1}{1+x^2}dx = dz$

$\therefore \int \frac{\tan^{-1} x}{1+x^2}dx = \int z dz = \frac{1}{2}z^2 + c$

$= \frac{1}{2}(\tan^{-1} x)^2 + c$

6(ii) Integrate: $\int \frac{e^{\tan^{-1} x}}{1+x^2}dx$; **[JB. 00; DB. 09]**

Solution: $\therefore \int \frac{e^{\tan^{-1} x}}{1+x^2}dx$

$= \int e^z dz$

$= e^{\tan^{-1} x} + c$

6(iii) Integrate: $\int \frac{(\tan^{-1} x)^2}{1+x^2}dx$

Solution: $\therefore \int \frac{(\tan^{-1} x)^2}{1+x^2}dx$

ধরি, $\tan^{-1} x = z$

$\Rightarrow \frac{1}{1+x^2}dx = dz$

$= \int z^2 dz$

$= \frac{1}{3}z^3 + c$

$= \frac{1}{3}(\tan^{-1} x)^3 + c$

6(iv) Integrate: $\int \frac{dx}{(1+x^2)\tan^{-1} x}$ **[SB.11; DB. 10; BB. 04]**

Solution: $\therefore \int \frac{dx}{(1+x^2)\tan^{-1} x}$

ধরি, $\tan^{-1} x = z$

$\Rightarrow \frac{1}{1+x^2}dx = dz$

$= \int \frac{dz}{z}$

$= \ln |z| + c$

$= \ln |\tan^{-1} x| + c$

6(v) Integrate: $\int \frac{dx}{(1+x^2)(1+\tan^{-1} x)}$

Solution:

$\therefore \int \frac{dx}{(1+x^2)(1+\tan^{-1} x)}$

ধরি, $1 + \tan^{-1} x = z$

$\Rightarrow 0 + \frac{1}{1+x^2}dx = dz$

$\therefore \frac{dx}{1+x^2} = dz$

$= \int \frac{dz}{z}$

$= \ln |z| + c$

$= \ln |1 + \tan^{-1} x| + c$

6(vi) Integrate: $\int \frac{x^2 \tan^{-1} x^3}{1+x^6}dx$

[CB, 08; JB, 06; CtgB. 07; MB, 08]

Solution:

$\int \frac{x^2 \tan^{-1} x^3}{1+x^6}dx$

ধরি, $\tan^{-1} x^3 = z$

$\Rightarrow \frac{1}{1+(x^3)^2}3x^2 dx = dz$

$\therefore \frac{x^2 dx}{1+x^6} = \frac{1}{3}dz$

$= \frac{1}{3}\int z dz$

$= \frac{1}{3} \cdot \frac{1}{2}z^2 + c$

$= \frac{1}{6}(\tan^{-1} x^3)^2 + c$

7(i) Integrate: $\int \frac{dx}{\sqrt{9-16x^2}}$; **[DB. 04,06; RB, 03,06]**

Solution: $\int \frac{dx}{\sqrt{9-16x^2}} = \int \frac{dx}{\sqrt{16\left(\frac{9}{16}-x^2\right)}}$

$$=\frac{1}{4}\int\frac{dx}{\sqrt{\left(\frac{3}{4}\right)^2-x^2}}=\frac{1}{4}\sin^{-1}\left(\frac{x}{\frac{3}{4}}\right)+c$$

$$=\frac{1}{4}\sin^{-1}\left(\frac{4x}{3}\right)$$

7(ii) Integrate: $\int\frac{\mathbf{dx}}{\sqrt{\mathbf{5-4x^2}}}$ **;[CB. 12; JB. 11; Ctg B.B 03]**

Solution: $\int\frac{dx}{\sqrt{5-4x^2}}=\int\frac{dx}{\sqrt{4\left(\frac{5}{4}-x^2\right)}}$

$=\frac{1}{2}\int\frac{dx}{\sqrt{\left(\frac{\sqrt{5}}{2}\right)^2-x^2}}=\frac{1}{2}\sin^{-1}\left(\frac{x}{\frac{\sqrt{5}}{2}}\right)+c$

7(iii) Integrate: $\int\frac{\boldsymbol{dx}}{\sqrt{\mathbf{2-3}\boldsymbol{x}^{\mathbf{2}}}}$

[BB.SB. 13; CB. 14, 10, 07; JB, 05]

Solution: $\int\frac{dx}{\sqrt{2-3x^2}}=\int\frac{dx}{\sqrt{3\left(\frac{2}{3}-x^2\right)}}$

$$=\frac{1}{\sqrt{3}}\int\frac{dx}{\sqrt{\left(\sqrt{\frac{2}{3}}\right)^2-x^2}}$$

$$=\frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{x}{\sqrt{\frac{2}{3}}}\right)+c=\frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2}}\right)+c$$

7(iv) Integrate: $\int\frac{\boldsymbol{dx}}{\sqrt{\mathbf{2-3}\boldsymbol{x}^{\mathbf{2}}}}$ **[BB.SB. 13; CB. 14, 10]**

Solution: $\int\frac{dx}{\sqrt{2-3x^2}}=\int\frac{dx}{\sqrt{3\left(\frac{2}{3}-x^2\right)}}$

$$=\frac{1}{\sqrt{3}}\int\frac{dx}{\sqrt{\left(\sqrt{\frac{2}{3}}\right)^2-x^2}}$$

$$=\frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{x}{\sqrt{\frac{2}{3}}}\right)+c=\frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2}}\right)+c$$

8(i) Integratet $\int\frac{\boldsymbol{dx}}{\mathbf{16}+\boldsymbol{x}^{\mathbf{2}}}$

Solution: $\int\frac{dx}{16+x^2}=\int\frac{dx}{4^2+x^2}=\frac{1}{4}\tan^{-1}\frac{x}{4}+c$

8(ii) Integrate: $\int\frac{\boldsymbol{dx}}{\mathbf{9}+\boldsymbol{x}^{\mathbf{2}}}$

Solution: $\int\frac{dx}{9+x^2}=\int\frac{dx}{3^2+x^2}-\frac{1}{3}\tan^{-1}\frac{x}{3}+c$

8(iii) lategrate: $\int\frac{\boldsymbol{dx}}{\mathbf{4}\boldsymbol{x}^{\mathbf{2}}+\mathbf{9}}$

Solution: $\int\frac{dx}{4x^2+9}-\int\frac{dx}{4\left(x^2+\frac{9}{4}\right)}=\frac{1}{4}\int\frac{dx}{x^2+\left(\frac{3}{2}\right)^2}$

$$=\frac{1}{4}\frac{1}{\frac{3}{2}}\tan^{-1}\left(\frac{x}{\frac{3}{2}}\right)+c=\frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)+c$$

9(i) Integrate: $\int\frac{\boldsymbol{dx}}{\boldsymbol{x}^{\mathbf{2}}-\mathbf{16}}$

Solution: $\int\frac{dx}{x^2-16}=\int\frac{dx}{x^2-4^2}$

$$=\frac{1}{2.4}\ln\left|\frac{x-4}{x+4}\right|+c=\frac{1}{8}\ln\left|\frac{x-4}{x+4}\right|+c$$

9(ii) Integrate: $\int\frac{\boldsymbol{dx}}{\mathbf{9}\boldsymbol{x}^{\mathbf{2}}-\mathbf{16}}$; **[DB. 03]**

Solation: $\int\frac{dx}{9x^2-16}=\int\frac{dx}{9\left(x^2-\frac{16}{9}\right)}=\frac{1}{9}\int\frac{dx}{x^2-\left(\frac{4}{3}\right)^2}$

$$=\frac{1}{9.2\cdot\frac{4}{3}}\ln\left|\frac{x-\frac{4}{3}}{x+\frac{4}{3}}\right|+c=\frac{1}{24}\ln\left|\frac{3x-4}{3x+4}\right|+c$$

9(iii) Integrate: $\int\frac{\mathbf{dx}}{\mathbf{9x^2-16}}$**; [DB. 03]**

Solution: $\int\frac{dx}{9x^2-16}=\int\frac{dx}{9\left(x^2-\frac{16}{9}\right)}=\frac{1}{9}\int\frac{dx}{x^2-\left(\frac{4}{3}\right)^2}$

$$=\frac{1}{9.2\cdot\frac{4}{3}}\ln\left|\frac{x-\frac{4}{3}}{x+\frac{4}{3}}\right|+c=\frac{1}{24}\ln\left|\frac{3x-4}{3x+4}\right|+c$$

9(iv) Integrate: $\int\frac{\boldsymbol{dx}}{\mathbf{16-4}\boldsymbol{x}^{\mathbf{2}}}$; **[JB. 00; BB. 13]**

Solution: $\int\frac{dx}{16-4x^2}=\int\frac{dx}{4(4-x^2)}=\frac{1}{4}\int\frac{dx}{2^2-x^2}$

$$=\frac{1}{4}\cdot\frac{1}{2.2}\ln\left|\frac{2+x}{2-x}\right|+c=\frac{1}{16}\ln\left|\frac{2+x}{2-x}\right|+c$$

10(i) Integrate: $\int\frac{\boldsymbol{dx}}{\boldsymbol{x}^{\mathbf{2}}+\mathbf{6}\boldsymbol{x}+\mathbf{25}}$ **; [JB.12]**

Solution: $\int\frac{dx}{x^2+6x+25}-\int\frac{dx}{(x+3)^2+25-9}$

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$$= \int \frac{dx}{(x+3)^2+4^2} = \frac{1}{4}\tan^{-1}\left(\frac{x+3}{4}\right)+c$$

10(ii) Integrate: $\int \frac{dx}{x^2-6x+18}$

Solution: $\int \frac{dx}{x^2-6x+18} = \int \frac{dx}{(x-3)^2+18-9}$

$$= \int \frac{dx}{(x-3)^2+3^2} = \frac{1}{3}\tan^{-1}\left(\frac{x-3}{3}\right)+c$$

10(iii) Integrate: $\int \frac{dx}{x^2-x+1}$; [**Ctg.03]**

Solution: $\int \frac{dx}{x^2-x+1} = \int \frac{dx}{\left(x-\frac{1}{2}\right)^2+1-\frac{1}{4}}$

$$= \int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}+c$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}}+c$$

10(iv) Integrate: $\int \frac{dx}{x^2+x}$;

Solution: $\int \frac{dx}{x^2+x} = \int \frac{dx}{x^2+2\cdot x\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$

$$= \int \frac{dx}{\left(x+\frac{1}{2}\right)^2-\frac{1}{4}} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{2\cdot\frac{1}{2}}\ln\left|\frac{x+\frac{1}{2}-\frac{1}{2}}{x+\frac{1}{2}+\frac{1}{2}}\right|+c = \ln\left|\frac{x}{x+1}\right|+c$$

10(v) Integrate: $\int \frac{dx}{5+4x-x^2}$ **[KUET, 04 − 05]**

Solution: $\int \frac{dx}{5+4x-x^2} = \int \frac{dx}{9-x^2+4x-4}$

$$= \int \frac{dx}{9-(x-2)^2} = \int \frac{dx}{3^2-(x-2)^2}$$

$$= \frac{1}{2.3}\ln\left|\frac{3+(x-2)}{3-(x-2)}\right|+c = \frac{1}{6}.\ln\left|\frac{1+x}{5-x}\right|+c$$

11(i) Integrate: $\int \frac{3x^2}{1+x^6}dx$ **[SB. 12; CtgB. 08]**

Solution: ধরি, $x^3 = z \Rightarrow 3x^2dx = dz$

$$\int \frac{3x^2}{1+x^6}dx = \int \frac{3x^2}{1+(x^3)^2}dx = \int \frac{dz}{1+z^2}$$

$$= \tan^{-1}z+c = \tan^{-1}(x^3)+c$$

11(ii) Integrate: $\int \frac{xdx}{x^4+1}$ **[BB. 11; RB. 08]**

Solution: ধরি, $x^2 = z$

$$\Rightarrow 2xdx = dz \therefore xdx = \frac{1}{2}dz$$

$$\int \frac{xdx}{x^4+1} = \int \frac{xdx}{(x^2)^2+1} = \frac{1}{2}\int \frac{dz}{z^2+1}$$

$$= \frac{1}{2}\tan^{-1}z+c$$

$$= \frac{1}{2}\tan^{-1}(x^2)+c$$

11(iii)Integrate: $\int \frac{\cos x\,dx}{3+\cos^2 x}$

[KUET. 05-06; BUTEX.06-07]

Solution: $\int \frac{\cos x}{3+\cos^2 x}dx$

$=\int \frac{\cos x dx}{3+1-\sin^2 x}$

$=\int \frac{\cos x\,dx}{4-\sin^2 x}$ Let, $\sin x = z$

$\therefore \cos x\,dx = dz$

$=\int \frac{dz}{4-z^2}$

$$= \int \frac{dz}{2^2-z^2} = \frac{1}{2.2}\ln\left|\frac{2+z}{2-z}\right|+c = \frac{1}{4}\ln\left|\frac{2+\sin x}{2-\sin x}\right|+c$$

11(iv) Integrate: $\int \frac{5e^{2x}}{1+e^{4x}}dx$; $[\text{Ctg}, 01]$

Solution: ধরি, $e^{2x} = z \Rightarrow 2e^{2x}dx = dz$

$$\therefore e^{2x}dx = \frac{1}{2}$$

$$\therefore \int \frac{5e^{2x}}{1+e^{4x}}dx = 5\int \frac{e^{2x}dx}{1+(e^{2x})^2}$$

$$=\frac{5}{2}\int\frac{dz}{1+z^2}=\frac{5}{2}\tan^{-1} z+c$$
$$=\frac{5}{2}\tan^{-1}(e^{2x})+c$$

12(i) Integrate: $\int\frac{x^2dx}{\sqrt{1-x^6}}$

[RB. 12; DjB. 12; JB. 11]

Solution: ধরি, $x^3=z\Rightarrow 3x^2dx=dz$

$$\therefore x^2dx=\frac{1}{3}dz$$

$$\therefore\int\frac{x^2dx}{\sqrt{1-x^6}}=\int\frac{x^2dx}{\sqrt{1-(x^3)^2}}$$

$$=\frac{1}{3}\int\frac{dz}{\sqrt{1-z^2}}=\frac{1}{3}\sin^{-1} z+c$$

$$=\frac{1}{3}\sin^{-1}(x^3)+c$$

12(ii) Integrate: $\int\frac{dx}{x\sqrt{x^4-1}}$ **[RB. 11; JB.01]**

Solution: $\int\frac{dx}{x\sqrt{x^4-1}}$ ধরি, $x^2=z\Rightarrow 2xdx=dz$

$=\int\frac{2xdx}{2x^2\sqrt{x^4-1}}=\frac{1}{2}\int\frac{dz}{z\sqrt{z^2-1}}$

$=\frac{1}{2}\sec^{-1}z+e$

[Formula: $\int\frac{dx}{x\sqrt{x^2-1}}=\sec^{-1} x+c$]

$$=\frac{1}{2}\sec^{-1} x^2+c$$

13) Integrate: $\int\frac{xdx}{\sqrt{1-x}}$ **; [DB.14; CtgB. 14]**

Solution: $\int\frac{xdx}{\sqrt{1-x}}$ ধরি, $\sqrt{1-x}=z.$

$$1-x=z^2$$
$$x=1-z^2$$
$$dx=-2zdz$$

$$=\int\frac{(1-z^2)\cdot(-2zdz)}{z}$$

$$=-2\int(1-z^2)dz$$

$$=-2\left(z-\frac{z^3}{3}\right)+c=-2z\left(1-\frac{z^2}{3}\right)+c$$

$$=-2\sqrt{1-x}\left(1-\frac{1-x}{3}\right)+c$$

$$=-\frac{2}{3}\sqrt{1-x}(x+2)+c$$

14)Integrate: $\int\sin^{-1}\sqrt{\frac{x}{a+x}}dx$

Solution: ধরি, $I=\int\sin^{-1}\sqrt{\frac{x}{a+x}}dx$

এবং $x=a\tan^2\theta\ \therefore\theta=\tan^{-1}\sqrt{\frac{x}{a}}$

$dx=2a\tan\theta\sec^2\theta\, d\theta$

$$I=\int\left(\sin^{-1}\sqrt{\frac{a\tan^2\theta}{a(1+\tan^2\theta)}}\right)2a\tan\theta\sec^2\theta d\theta$$

$$=\int\sin^{-1}\left(\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}\right)2a\tan\theta\sec^2\theta\, d\theta$$

$$=\int\sin^{-1}(\sin\theta)\,2a\tan\theta\sec^2\theta\, d\theta$$

$$\therefore I=\int\theta\, 2a\tan\theta\sec^2\theta d\theta\ \text{....................(i)}$$

$$=2a\left[\theta\tan\theta\int\sec^2\theta d\theta\right.$$
$$\left.-\int\left[\frac{d}{d\theta}(\theta\tan\theta)\int\sec^2\theta d\theta\right)d\theta\right]$$

$$=2a\left[\theta\tan^2\theta-\int(\theta\sec^2\theta+\tan\theta)\tan\theta\right]d\theta$$

$$=2a\theta\tan^2\theta-\int 2a\,\theta\tan\theta\sec^2\theta\, d\theta$$
$$-2a\int\tan^2\theta d\theta$$

$$=2a\theta\tan^2\theta-I-2a\int(\sec^2\theta-1)d\theta$$

$$\therefore I+I=2a\theta\tan^2\theta-2a(\tan\theta-\theta)$$

or, $2I=2a\theta\tan^2\theta-2a(\tan\theta-\theta)$

or, $I=a\theta\tan^2\theta-a(\tan\theta-\theta)$

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or,I= $a\tan^{-1}\left(\sqrt{\frac{x}{a}}\right)\frac{x}{a} - a\left(\sqrt{\frac{x}{a}} - \tan^{-1}\sqrt{\frac{x}{a}}\right) + c$

$or, I = x\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$

$$\therefore I = (x+a)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + c$$

বিশেষ কত গুল অংক(বিশেষ গ্রুপ ১) [su,10]

$\int \frac{dx}{x^{\frac{1}{a}} - x^{\frac{1}{b}}}$ এবং $\int \frac{x^{\frac{1}{a}}dx}{1+x^{\frac{1}{b}}}$ আকারের যোগজ এর a ও b এর ল সা গু c হলে $x = z^c$ ধরে অংক করতে হবে।

Ques:15(i) $\int \frac{dx}{x^{\frac{1}{2}} - x^{\frac{1}{4}}}$

Solution: $\int \frac{dx}{x^{\frac{1}{2}} - x^{\frac{1}{4}}}$

ধরি, $x = z^4$
$\therefore dx = 4z^3dz$
এবং, $z^2 = \sqrt{x}$
$\therefore x^{\frac{1}{4}} = z$

$= \int \frac{4z^3dz}{z^2 - z}$

$= 4\int \frac{z^2dz}{z-1}$

$= 4\int \frac{(z^2-1)+1}{z-1}dz$

$= 4\int zdz + 4\int dz + 4\int \frac{dz}{z-1}$

$= 4\cdot\frac{z^2}{2} + 4z + 4\cdot\ln|z-1| + c$

$= 2\sqrt{x} + 4\sqrt[4]{x} + 4\ln|\sqrt[4]{x} - 1| + c$ (Ans.)

Ques:15(ii) $\int \frac{\sqrt{x}dx}{1+\sqrt[3]{x}}$

Solution $\int \frac{\sqrt{x}dx}{1+\sqrt[3]{x}}$

ধরি $x = z^6$
$\therefore dx = 6z^5dz$
এবং $x^{\frac{1}{2}} = z^3$
$\therefore x^{\frac{1}{6}} = z$

$= \int \frac{x^{\frac{1}{2}}dx}{1+x^{\frac{1}{3}}}$

$= \int \frac{z^3\cdot 6z^5dz}{1+z^2}$

$= 6\int \frac{z^8}{1+z^2}dz$

$= 6\int \frac{z^6(z^2+1) - z^4(z^2+1) + z^2(z^2+1) - 1\cdot(z^2+1) + 1}{1+z^2}dz$

$= 6\int \left(z^6 - z^4 + z^2 - 1 + \frac{1}{1+z^2}\right)dz$

$= 6\int z^6dz - 6\int z^4dz + 6\int z^2dz - 6\int 1dz + 6\int \frac{1}{1+z^2}dz$

$= \frac{6}{7}z^7 - \frac{6}{5}z^5 + 2z^3 - 6z + 6\tan^{-1}z + c$

$= \frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2x^{\frac{1}{2}} - 6x^{\frac{1}{6}} + 6\tan^{-1}\left(x^{\frac{1}{6}}\right) + c$ (Ans.)

(**বিশেষ গ্রুপ ২**)

(লবকে$\sqrt{}$ মুক্ত করতে হবে)

Ques $-$**16(i)** $\sqrt{\frac{1+x}{1-x}}dx$ **[KUET, '11-12]**

Solution: $\int \sqrt{\frac{1+x}{1-x}}dx$

$I = \int \sqrt{\frac{1+x}{1-x}}dx$

$= \int \sqrt{\frac{(1+x)(1+x)}{(1-x)(1+x)}}dx$

$= \int \frac{1+x}{\sqrt{1-x^2}}dx$

$= \int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{xdx}{\sqrt{1-x^2}}$

$= \sin^{-1}x + I_1$ (ধরি)

ধরি, $1 - x^2 = z$
$-$ 2x dx = dz
X dx = $\frac{-dz}{2}$

$\therefore I_1 = \int \frac{xdx}{\sqrt{1-x^2}}$

$= \int \frac{\frac{-dz}{2}}{\sqrt{z}}$

$= -\int \frac{dz}{2\sqrt{z}}$

$= -\sqrt{z}$

$= -\sqrt{1-x^2}$

$\therefore I = \sin^{-1} x - \sqrt{1-x^2} + c$

16(ii) $\int \sqrt{\dfrac{1-x}{1+x}}\,dx$

Solution $\int \sqrt{\frac{1-x}{1+x}}\,dx$

$= \int \sqrt{\dfrac{(1-x)(1-x)}{(1+x)(1-x)}}\,dx$

$= \int \sqrt{\dfrac{(1-x)^2}{1-x^2}}\,dx$

$= \int \dfrac{1-x}{\sqrt{1-x^2}}\,dx$

$= \int \dfrac{dx}{\sqrt{1-x^2}} - \int \dfrac{xdx}{\sqrt{1-x^2}}$

$= \sin^{-1} x - \int \dfrac{xdx}{\sqrt{1-x^2}}$

ধরি, $\sqrt{1-x^2} = z$
$1-x^2 = z^2$
$-$ 2x dx = 2z dz
X dx = −z dz

$= \sin^{-1} x + \int \dfrac{zdz}{z}$

$= \sin^{-1} x + \int dz$

$= \sin^{-1} x + z + c$

$= \sin^{-1} x + \sqrt{1-x^2} + c$ (Ans.)

16(iii) Integrate: $\int \sqrt{\frac{a+x}{x}}\,dx$

$= \int \dfrac{a+x}{\sqrt{(a+x)x}}\,dx$

$= \int \dfrac{a+x}{\sqrt{ax+x^2}}\,dx$

$= \dfrac{1}{2}\int \dfrac{2a+2x}{\sqrt{ax+x}}\,dx$

$= \dfrac{1}{2}\int \dfrac{a+2x+a}{\sqrt{ax+x^2}}\,dx$

$\mathrm{I} = \dfrac{1}{2}\int \dfrac{a+2x}{\sqrt{ax+x^2}}\,dx + \dfrac{1}{2}\int \dfrac{a}{\sqrt{ax+x^2}}\,dx$

or, $\mathrm{I} = \frac{1}{2} I_1 + \frac{1}{2} I_2$ (ধরি)

এখানে, $I_1 = \int \dfrac{a+2x}{\sqrt{ax+x^2}}\,dx$ ধরি, $ax + x^2 = z$

$= \int \dfrac{1}{\sqrt{z}}\,dz$ $(a+2x)dx = dz$

$= 2\int \dfrac{1}{2\sqrt{z}}\,dz = 2\sqrt{z} + c = 2\sqrt{ax+x^2} + c$

এবং $I_2 = \int \frac{1}{\sqrt{ax+x^2}}$

$= \int \frac{a}{\sqrt{x^2+2x\frac{a}{2}+\left(\frac{a}{2}\right)^2-\left(\frac{a}{2}\right)^2}}$

$= a\int \dfrac{1}{\sqrt{\left(x+\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2}}\,dx$

$= a \ln \left|\sqrt{\left(x+\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2} + \left(x+\frac{a}{2}\right)\right|$

$\left[\text{Formula: } \int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$\therefore I = \frac{1}{2}\cdot 2\sqrt{ax+x^2} + \frac{1}{2}\cdot a\ln\left|\sqrt{\left(x+\frac{a}{2}\right)^2 - \left(\frac{a}{2}\right)^2} + \left(x+\frac{a}{2}\right)\right|$

$= \sqrt{ax+x^2} + \frac{a}{2}\ln\left|\sqrt{ax+x^2} + \left(x+\frac{a}{2}\right)\right| + c$

(**বিশেষ গ্রুপ ৩**) **[su,9,14]**

$\int \frac{dx}{\sin^p x \cos^q x}$ এবং $\int \frac{dx}{\sin^p x + \cos^q x}$ আকারের যোগজ এর ক্ষেত্রে p + q =m জোড় সংখ্যা হলে $\sec^m x$ দ্বারা লব ও হরকে গুন করতে হবে এবং $\tan x = z$ ধরে অংক করতে হবে।

Ques 17(i): $\int \dfrac{dx}{1+\cos^2 x}$

Solution: $\int \frac{dx}{1+\cos^2 x}$

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$= \int \frac{\sec^2 x dx}{\sec^2 x + 1}$

$= \int \frac{\sec^2 x dx}{1 + \tan^2 x + 1}$

$= \int \frac{\sec^2 x dx}{2 + \tan^2 x}$

$= \int \frac{dz}{(\sqrt{2})^2 + z^2}$

ধরি, $\tan \boldsymbol{x} = z$

$\therefore \sec^2 \boldsymbol{x}\, \boldsymbol{dx} = dz$

$= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{z}{\sqrt{2}}\right) + c$

$= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan x}{\sqrt{2}}\right) + c$ (Ans.)

Ques17(ii) : $\int \frac{d\theta}{1 + 3\cos^2 \theta}$

solution $\int \frac{d\theta}{1+3\cos^2 \theta}$

$= \int \frac{\sec^2 \theta d\theta}{\sec^2 \theta (1 + 3\cos^2 \theta)}$

$\sec^2\boldsymbol{\theta}$ দ্বারা লব ও হরকে গুন করি

$= \int \frac{\sec^2 \theta d\theta}{\sec^2 \theta + 3}$

$= \int \frac{\sec^2 \theta d\theta}{1 + \tan^2 \theta + 3}$

$= \int \frac{\sec^2 \theta d\theta}{4 + \tan^2 \theta}$

ধরি, $\tan \boldsymbol{\theta} = z$

$\therefore \sec^2 \boldsymbol{\theta} \boldsymbol{d\theta} = dz$

$= \int \frac{dz}{2^2 + z^2}$

$= \frac{1}{2} \tan^{-1} \frac{z}{2} + c$

$= \frac{1}{2} \tan^{-1} \left(\frac{1}{2} \tan \theta\right) + c$ (Ans.)

Ques 17 (iii) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} \boldsymbol{dx}$

$\sec^2 x$ দ্বারা লব ও হরকে গুন করি

$= \int \frac{\sqrt{\tan x}\, \sec^2 x\; dx}{\sin x\; \cos x \sec^2 x}$

$= \int \frac{\sqrt{\tan x}\, \sec^2 x\, dx}{\tan x}$

$= \int \frac{\sec^2 x dx}{\sqrt{\tan x}}$

ধরি, $\tan \boldsymbol{x} = z$

$\therefore \sec^2 \boldsymbol{x}\, \boldsymbol{dx} = dz$

$= \int \frac{dz}{\sqrt{z}}$

$= 2\sqrt{z} + c$

$= 2\sqrt{\tan x} + c$

Ques 17 (iv) $\int \frac{1 + \tan \frac{x}{2}}{1 + \sin x} \boldsymbol{dx}$

$= \int \frac{1 + \tan \frac{x}{2}}{1 + 2\sin \frac{x}{2} \cos \frac{x}{2}} dx$

$= \int \frac{\left(1 + \tan \frac{x}{2}\right) \sec^2 \frac{x}{2}}{\sec^2 \frac{x}{2} + 2\tan \frac{x}{2}} dx$

$= \int \frac{\left(1 + \tan \frac{x}{2}\right) \sec^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} + 2\tan \frac{x}{2}} dx$

ধরি, $1 + \tan \frac{x}{2} = z$

$\therefore \sec^2 \frac{x}{2} \boldsymbol{dx} = 2dz$

$$= \int \frac{\left(1+\tan\frac{x}{2}\right)\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{1+\tan\frac{x}{2}}$$

$$= \int \frac{2dz}{z} = 2\ln z + c$$

$$= 2\ln\left|1+\tan\frac{x}{2}\right| + c$$

18(i) $\int \frac{dx}{\sqrt{\sin x}\sqrt{\cos^7 x}}$

$$= \int \frac{\sec^4 xdx}{\sqrt{\tan x}}$$

$$= \int \frac{(1+\tan^2 x)\sec^2 xdx}{\sqrt{\tan x}}$$

$$= \int \frac{(1+z^2)dz}{\sqrt{z}}$$

$$= \int z^{-\frac{1}{2}}dz + \int z^{\frac{3}{2}}dz$$

$$= 2\sqrt{z} + \frac{2}{5}z^{5/2} + c$$

$$= 2\sqrt{(tanx)} + \frac{2}{5}(tanx)^{5/2} + c$$

18(ii) $\int \frac{dx}{\sqrt{\sin^3 x}\sqrt{\cos^5 x}}$

$$= \int \frac{\sec^4 xdx}{(\tan x)^{3/2}}$$

$$= \int \frac{(1+\tan^2 x)\sec^2 xdx}{(\tan x)^{3/2}}$$

$$= \int \frac{(1+z^2)dz}{z^{\frac{3}{2}}}$$

ধরি, $\tan x = z$

$\therefore \sec^2 x\,dx = dz$

$$= \int z^{-\frac{3}{2}}dz + \int z^{\frac{1}{2}}dz$$

$$= \frac{-2}{\sqrt{\tan x}} + \frac{2}{3}(\tan x)^{3/2} + c.$$

ধরি, $\tan x = z$

$\therefore \sec^2 x\,dx = dz$

(বিশেষ গ্রুপ ৪)

[su19]

$\int \frac{p\cos x + q\sin x}{a\cos x + b\sin x}dx$ **আকারের ইন্টিগ্রালের জন্য,**

লব = L (হর) +M(হরের অন্তরক সহগ), অতঃপর sin x, cos x এর সহগ ও ধ্রুবপদ সমীকৃত করে L, M এর মান নির্ণয় করতে হবে ।

$\int \frac{p\sin x + q\cos x + r}{a\cos x + b\sin x + c}dx$ **আকারের ইন্টিগ্রালের জন্য,**

লব =L (হর) +M (হরের অন্তরক সহগ) + N ধরতে হবে, অতঃপর sin x, cos x এর সহগ ও ধ্রুবপদ সমীকৃত করে L, M, N এর মান নির্ণয় করতে হবে ।

example. $\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x}dx = ?$

Sol": $\int \frac{a\sin x + b\cos x}{c\sin x + d\cos x}dx$

$$= \int \frac{M(c\sin x + d\cos x) + N(c\cos x - d\sin x)}{c\sin x + d\cos x}dx$$

$$= M\int dx + N\int \frac{c\cos x - d\sin x}{c\sin x + d\cos x}dx$$

$$= Mx + N\ln|c\sin x + d\cos x|$$

যেখানে, $Mc - Nd = a$

$Md + Nc = b$

$\therefore M = \frac{ac+bd}{c^2+d^2}; N = \frac{bc-ad}{c^2+d^2}$

Ques19(i) $\int \frac{1}{1+\tan x}dx$

[SB'14;R.B.'12,'08:J.B.'12]

Solution $\int \frac{dx}{1+\tan x} = \int \frac{dx}{1+\frac{\sin x}{\cos x}}$

$$= \int \frac{dx}{\frac{\cos x+\sin x}{\cos x}} = \int \frac{\cos x}{\sin x+\cos x}dx$$

$$= \frac{1}{2}\int \frac{2\cos x}{\sin x+\cos x}dx$$

$$= \frac{1}{2}\int \frac{(\sin x+\cos x)+(\cos x-\sin x)}{\sin x+\cos x}dx$$

$$= \frac{1}{2}\left\{\int dx+\int \frac{\cos x-\sin x}{\sin x+\cos x}dx\right\}$$

$= \frac{1}{2}\{x+\ln |\sin x+\cos x|\}+c$ (Ans.)

Ques19(ii) $\int \frac{dx}{2+\cot x}$

Solution $\int \frac{dx}{2+\cot x}$

$$= \int \frac{dx}{2+\frac{\cos x}{\sin x}} = \int \frac{\sin x\, dx}{2\sin x+\cos x}$$

$$= \frac{1}{5}\cdot\int \frac{5\sin x dx}{2\sin x+\cos x}$$

$$= \frac{1}{5}\int \frac{2(2\sin x+\cos x)-(2\cos x-\sin x)}{2\sin x+\cos x}$$

$$= \frac{2}{5}\int dx-\frac{1}{5}\int \frac{(2\cos x-\sin x)}{(2\sin x+\cos x)}dx$$

$= \frac{2x}{5}-\frac{1}{5}\ln |2\sin x+\cos x|+c$ (Ans.)

Ques19(iii) $\int \frac{3\sin x dx}{\cos x+\sin x}$

Solution $\int \frac{3\sin x dx}{\cos x+\sin x}$

$$= \frac{1}{2}\cdot\int \frac{3\times 2\sin x dx}{\cos x+\sin x}$$

$$= \frac{1}{2}\int \frac{3(\cos x+\sin x)-3(\cos x-\sin x)}{\cos x+\sin x}$$

$$= \frac{3}{2}\int dx-\frac{3}{2}\int \frac{\cos x-\sin x}{\cos x+\sin x}dx$$

$= \frac{3x}{2}-\frac{3}{2}\ln |\cos x+\sin x|+c$ (Ans.)

$$\left[\therefore \int \frac{f'(x)}{f(x)}dx = \ln |f(x)|+c\right]$$

(বিশেষ গ্রুপ ৫)   [su 18]

$\int\frac{dx}{a+b\sin x}$, $\int\frac{dx}{a+b\cos x}$, $\int\frac{dx}{a\sin x+b\cos x+c}$ জাতীয় ইন্টিগ্রেশন ।

* উপরোক্ত আকারের ইনটিগ্রালে $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ এবং $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ বসাতে হয়, ত্রিকোণমিতিক Function হরে (sinx, cosx) থাকলে $\tan^2\frac{x}{2}$ তে প্রকাশ করতে হয় । কারণ $\tan\frac{x}{2}=z$ ধরলে লবে $\sec^2\frac{x}{2}dx = 2dz$ হয় এবং আদর্শ বীজগাণিতীয় আকারে প্রকাশ হয় ।

Ques20(i) $\int \frac{dx}{1+\sin x-\cos x}$ **[BUET.'11-12]**

Solution $\int \frac{dx}{1+\sin x-\cos x}$

$$= \int \frac{dx}{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}-\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}-1+\tan^2\frac{x}{2}}$$

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$$= \int \frac{\sec^2 \frac{x}{2} dx}{2\tan^2 \frac{x}{2} + 2\tan \frac{x}{2}}$$

ধরি, , $z = \tan \frac{x}{2}$

$\Rightarrow dz = \frac{1}{2}\sec^2 \frac{x}{2} dx$

$\Rightarrow 2dz = \sec^2 \frac{x}{2} dx$

$$= \int \frac{2dz}{2z^2+2z}$$

$$= \int \frac{dz}{z^2 + z}$$

$$= \int \frac{dz}{z^2 + 2\cdot z\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$= \int \frac{dz}{\left(z+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{2\cdot\frac{1}{2}} \ln \left|\frac{z+\frac{1}{2}-\frac{1}{2}}{z+\frac{1}{2}+\frac{1}{2}}\right| + c$$

$$= \ln \left|\frac{z}{z+1}\right| + c$$

$$= \ln \left|\frac{\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right| + c \text{ (Ans.)}$$

20(ii) $\int \frac{dx}{2+\sin x} = \int \frac{dx}{2+\frac{5\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{2\left(1+\tan^2\frac{x}{2}\right) + 2\tan\frac{x}{2}}$$

$$= \frac{1}{2}\int \frac{\sec^2\frac{x}{2}dx}{\tan^2\frac{x}{2}+\tan\frac{x}{2}+1}$$

ধরি, , $z = \tan\frac{x}{2}$

$\Rightarrow dz = \frac{1}{2}\sec^2 \frac{x}{2} dx$

$$= \int \frac{dz}{z^2 + 2z\frac{1}{2} + \frac{1}{4} + \left(1 - \frac{1}{4}\right)}$$

$$= \int \frac{dz}{\left(z+\frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\frac{z+\frac{1}{2}}{\frac{\sqrt{3}}{2}} + c$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan\frac{x}{2}+1}{\sqrt{3}}\right) + c$$

(বিশেষ গ্রুপ ৬)

$\int\frac{dx}{a\sin x + b\cos x}$ জাতীয় ইনটিগ্রালে ।

a=rcos α এবং b=r sin α ধরে অংক করতে হবে।

21) Integrate: $\int \frac{dx}{a\cos x + b\sin x}$

Solution: $\int \frac{dx}{a\cos x + b\sin x}$

$$= \int \frac{dx}{r(\sin\alpha\cdot\cos x + \cos\alpha\cdot\sin x)}$$

$$= \frac{1}{r}\int \frac{dx}{\sin(x+\alpha)}$$

$$= \frac{1}{r}\int \operatorname{cosec}(x+\alpha)dx$$

$$= \frac{1}{r}\ln\left|\tan\left(\frac{x+\alpha}{2}\right)\right| + c$$

ধরি, $a = r\sin\alpha$

$b = r\cos\alpha$

$\therefore r^2(\sin^2\alpha + \cos^2\alpha)$

$= a^2 + b^2$

$\therefore r = \sqrt{a^2+b^2}$

And $\frac{r\sin\alpha}{r\cos\alpha} = \frac{a}{b}$

$\Rightarrow \tan\alpha = \frac{a}{b}$

$\therefore \alpha = \tan^{-1}\frac{a}{b}$

$$= \frac{1}{\sqrt{a^2+b^2}}\ln\left|\tan\left(\frac{x+a}{2}\right)\right| + c$$

$$= \frac{1}{\sqrt{a^2+b^2}}\ln \mid \tan\frac{1}{2}\left(x + \tan^{-1}\frac{a}{b}\right)$$

(বিশেষ গ্রুপ ৭)

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$\int \frac{dx}{(ax+b)\sqrt{cx+d}}$, $\int (ax+b)\sqrt{cx+d}\ dx$, $\int \frac{(ax+b)}{\sqrt{cx+d}}dx$, $\int \frac{x\,dx}{(ax^2+b)\sqrt{cx^2+d}}$ জাতীয় ইনটিগ্রালে

$cx+d=z^2$ **ধরতে হবে।** **[su 17]**

21(i) Integrate: $\int \frac{dx}{(x-3)\sqrt{x+1}}$**; [DB. 10; SB. 13]**

Solution: ধরি, $x+1=z^2$

$\Rightarrow x-3=z^2-4 \Rightarrow dx=2zdz$

$$\therefore \int \frac{dx}{(x-3)\sqrt{x+1}}=\int \frac{2z\cdot dz}{(z^2-4)z}$$

$$=2\int \frac{dz}{z^2-4}$$

$$=2\cdot\frac{1}{2.2}\ln\left|\frac{z-2}{z+2}\right|+c=\frac{1}{2}\ln\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right|+c$$

21(ii) $\int \frac{xdx}{\sqrt{1-x}}$

ধরি, $z^2=1-x$ $\Rightarrow 2zdz=-dx$

$$=-\int \frac{2(1-z^2)zdz}{z}$$

$$=2\int (z^2-1)dz$$

$$=2\left(\frac{z^3}{3}-z\right)+c$$

$$=\frac{2}{3}(\sqrt{1-x})^3-2\sqrt{1-x}+c$$

$$=\frac{2}{3}(1-x)\sqrt{1-x}-2\sqrt{1-x}+c$$

$$=\frac{2}{3}\sqrt{1-x}(1-x-3)+c$$

$$=-\frac{2}{3}\sqrt{1-x}(x+2)+c$$

(বিশেষ গ্রুপ ৮)

$\int \frac{dx}{(cx+d)\sqrt{ax^2+bx+c}}$ জাতীয় ইনটিগ্রালে ।

$\frac{1}{cx+d}=z$ **ধরতে হবে।অর্থাৎ**

$cx+d=\frac{1}{z}$ **ধরতে হবে**

22. $\int \frac{dx}{(1+x)\sqrt{1-x^2}}$

ধরি, $1+x=\frac{1}{z}$ $\therefore x=\frac{1}{z}-1$ $\therefore\Rightarrow dx=-\frac{1}{z^2}dz$

Solution: $\int \frac{dx}{(1+x)\sqrt{1-x^2}}$

$$=\int \frac{-\frac{1}{z^2}dz}{\frac{1}{z}\sqrt{1-\left(\frac{1}{z}-1\right)^2}}$$

$$=-\int \frac{dz}{z}\cdot\frac{dz}{\sqrt{\frac{2}{z}-\frac{1}{z^2}}}$$

$$=-\int \frac{dz}{\sqrt{2z-1}}=-\frac{1}{2}\int \frac{2dz}{\sqrt{2z-1}}$$

$$=-\frac{1}{2}\cdot 2\sqrt{2z-1}+c$$

[Formula: $\int \frac{f'(x)}{\sqrt{f(x)}}dx=2\sqrt{f(x)}$ +c]

$$=-\sqrt{\frac{2}{1+x}-1}+c=-\sqrt{\frac{1-x}{1+x}}+c$$

(বিশেষ গ্রুপ ৯)

$\int \frac{dx}{(ax^2+b)\sqrt{cx^2+d}}$ জাতীয় ইনটিগ্রালে ।

প্রথমে, $x = \frac{1}{z}$ ধরে সরলীকরণ করে

$\sqrt{az^2+b} = u$ **ধরতে হবে।**

22(i) Integrate: $\int \frac{dx}{(x^2+1)\sqrt{x^2+4}}$

Solution: ধরি, $x = \frac{1}{z} \Rightarrow dx = -\frac{1}{z^2}dz$

$\therefore \int \frac{dx}{(x^2+1)\sqrt{x^2+4}}$

$$= \int \frac{-\frac{1}{z^2}dz}{\left(\frac{1}{z^2}+1\right)\sqrt{\frac{1}{z^2+4}}}$$

$$= -\int \frac{zdz}{(z^2+1)\sqrt{4z^2+1}}$$

ধরি, $\sqrt{4z^2+1} = u$
$\Rightarrow 4z^2+1 = u^2$
$\Rightarrow z^2 = \frac{u^2-1}{4}$
$\Rightarrow 2zdz = \frac{1}{4}\cdot 2udu$
$\therefore zdz = \frac{1}{4}udu$

$$= -\int \frac{\frac{1}{4}udu}{\left(\frac{u^2-1}{4}+1\right)u}$$

$$= -\int \frac{du}{u^2+3}$$

$$= -\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right) + c = -\frac{1}{\sqrt{3}}\tan^{-1}\frac{\sqrt{4z^2+1}}{\sqrt{3}} + c$$

$$= -\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{4+x^2}}{x\sqrt{3}}\right) + c$$

22(ii) Integrate: $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$

Solution: ধরি, $x = \frac{1}{z} \therefore dx = -\frac{1}{z^2}dz$

$$\therefore \int \frac{dx}{(1+x^2)\sqrt{1-x^2}} = \int \frac{-\frac{1}{z^2}dz}{\left(1+\frac{1}{z^2}\right)\sqrt{1-\frac{1}{z^2}}}$$

$$= -\int \frac{zdz}{(z^2+1)\sqrt{z^2-1}}$$

ধরি, $\sqrt{z^2-1} = u$
$\Rightarrow z^2-1 = u^2$
$\Rightarrow z^2 = u^2+1$
$\therefore zdz = udu$

$$= -\int \frac{udu}{(u^2+1+1)\cdot u}$$

$$= -\int \frac{du}{u^2+(\sqrt{2})^2}$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{2}}\right) + c$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{\frac{1}{x^2}}-1}{\sqrt{2}}\right) + c$$

$$= -\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x\sqrt{2}}\right) +$$

23. (i) Integrate: $\frac{x^2+1}{x^4+1}dx$ **[BUET. '14-15]**

Solution: $\int \frac{x^2+1}{x^4+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$

(x^2 দ্বারা লব ও হরকে ভাগ করি)

$$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}dx$$

$$= \int \frac{dz}{z^2+(\sqrt{2})^2}$$

ধরি, $x - \frac{1}{x} = z$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + c$$

$(1+\frac{1}{x^2})dx = dz$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c \text{ (Ans.)}$$

23(ii) Integrate: $\int \frac{dx}{(a^2+x^2)^{\frac{3}{2}}}$ **[JB. 02]**

Solution: ধরি, $x = a\tan\theta$

$\therefore dx = a\sec^2\theta\, d\theta$

$$\therefore \int \frac{dx}{(a^2+x^2)^{\frac{3}{2}}} = \int \frac{a\sec^2\theta\, d\theta}{(a^2+a^2\tan^2\theta)^{\frac{3}{2}}}$$

$$= \int \frac{a\sec^2\theta \cdot d\theta}{a^3(\sec^2\theta)^{\frac{3}{2}}} \int \frac{\sec^2\theta\, d\theta}{a^2\sec^3\theta}$$

$$= \frac{1}{a^2}\int \frac{d\theta}{\sec\theta} = \frac{1}{a^2}\int \cos\theta\, d\theta$$

$$= \frac{1}{a^2}\sin\theta + c = \frac{1}{a^2}\sin\left(\tan^{-1}\frac{x}{a}\right) + c$$

$$\left[\because x = a\tan\theta \Rightarrow \tan\theta = \frac{x}{a} \Rightarrow \theta = \tan^{-1}\frac{x}{a}\right]$$

$$= \frac{1}{a^2}\sin\left(\sin^{-1}\frac{x}{\sqrt{x^2+a^2}}\right) + c$$

$$= \frac{1}{a^2}\cdot\frac{x}{\sqrt{x^2+a^2}} + c$$

Exercise-10.4

বীজগাণিতীয় ভগ্নাংশকে আংশিক ভগ্নাংশে প্রকাশ করার সময় নিম্নোক্ত নিয়মগুলোর দিকে লক্ষ্য রাখবে

1. $\frac{x}{(x-2)(x-3)} \equiv \frac{A}{(x-2)} + \frac{B}{(x-3)}$
2. $\frac{x^2}{(x-2)(x-3)} \equiv 1 + \frac{A}{(x-2)} + \frac{B}{(x-3)}$
3. $\frac{x^3}{(x-2)(x-3)} \equiv 1 + \frac{A}{(x-2)} + \frac{B}{(x-3)}$
4. $\frac{x}{(x-2)^2(x-3)} \equiv \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)}$
5. $\frac{x}{(x^2-2)(x-3)} \equiv \frac{Ax+B}{(x^2-2)} + \frac{C}{(x-3)}$
6. $\frac{x}{(x^2-2)^2(x-3)} \equiv \frac{Ax+B}{(x^2-2)} + \frac{Cx+D}{(x^2-2)^2} + \frac{E}{(x-3)}$

Thumb's rule ঃ

Thumb's rule এর সাহায্যে সহজেই আংশিক ভগ্নাংশে পরিণত করা যায়।

যেমন–

$$\frac{1}{x^2-9} = \frac{1}{(x+3)(x-3)}$$

$$= \frac{1}{(x+3)(-3-3)} + \frac{1}{(3+3)(x-3)}$$

$$= \frac{1}{-6(x+3)} + \frac{1}{6(x-3)}$$

অর্থাৎ হরে যে পদ থাকবে সেটা = 0 যেমন ১ম ক্ষেত্রে(x+3=0 বা x= −3) থেকে x এর মান বের করে অন্যগুলোতে বসাতে হয়।

1(i) Integrate: $\int \frac{x+35}{x^2-25}dx$ [SB, 07; CigB. 04]

Solution: $\int \frac{x+35}{x^2-25}dx$

$$= \int \frac{x+35}{(x+5)(x-5)}dx$$

$$= \int \left[\frac{5+35}{(5+5)(x-5)} + \frac{-5+35}{(x+5)(-5-5)}\right]dx$$

$$= \int \left[\frac{40}{10\,(x-5)} + \frac{30}{(x+5)(-10)}\right]dx$$

$$= \int \left[\frac{4}{x-5} - \frac{3}{x+5}\right]dx$$

$$= 4\int \frac{1}{x-5}dx - 3\int \frac{1}{x+5}dx$$

$$= 4\ln|x-5| - 3\ln|x+5| + c$$

1(ii) Integrate: $\int \frac{1}{x(x-1)(x-3)}dx$

Solution: $\int \frac{1}{x(x-1)(x-3)}dx$

$$= \int \left[\frac{1}{x(0-1)(0-3)} + \frac{1}{1(x-1)(1-3)} + \frac{1}{3(3-1)(x-3)}\right]dx$$

$$= \int \left[\frac{1}{3x} - \frac{1}{2}\frac{1}{(x-1)} + \frac{1}{6}\frac{1}{(x-3)}\right]$$

$$= \frac{1}{3}\ln|x| - \frac{1}{2}\ln|x-1| + \frac{1}{6}\ln|x-3| + c$$

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2(i) Integrate: $\int \frac{x^2}{x^2-4}dx$ [SB. 01,08 RB. 04; BB, 04; CtgB. 02]

Solution: $\frac{x^2}{x^2-4} = 1 + \frac{4}{x^2-4} = 1 + \frac{4}{(x-2)(x+2)}$

$= 1 + \frac{4}{(x-2)(2+2)} + \frac{4}{(-2-2)(x+2)}$

$= 1 + \frac{1}{x-2} - \frac{1}{x+2}$

$\therefore \int \frac{x^2}{x^2-4}dx = \int dx + \int \frac{dx}{x-2} - \int \frac{dx}{x+2}$

$= x + \ln|x-2| - \ln|x+2| + c$

$= x + \ln\left|\frac{x-2}{x+2}\right| + c$

Alternative method: $\int \frac{x^2}{x^2-4}dx = \int \frac{x^2-4+4}{x^2-4}dx$

$= \int \left(\frac{x^2-4}{x^2-4} + \frac{4}{x^2-4}\right)dx$

$= \int \left(1 + \frac{4}{x^2-4}\right)dx$

$= \int 1dx + 4\int \frac{1}{x^2-2^2}dx$

$= x + 4 \times \frac{1}{2\times 2}\ln\left|\frac{x-2}{x+2}\right| + c$

$= x + \ln\left|\frac{x-2}{x+2}\right| + c$

2(ii) Integrate: $\int \frac{x^2-1}{x^2-4}dx$; [SB. 03, 05,12; DB. 15,11;CB. 01,09; JB. 09; BB. 13]

Solution: $\frac{x^2-1}{x^2-4} = 1 + \frac{3}{x^2-4} = 1 + \frac{3}{(x+2)(x-2)}$

$= 1 + \frac{3}{(2+2)(x-2)} + \frac{3}{(x+2)(-2-2)}$

$= 1 + \frac{3}{4}\frac{1}{x-2} - \frac{3}{4}\frac{1}{x+2}$

$\therefore \int \frac{x^2-1}{x^2-4}dx$

$= \int dx - \frac{3}{4}\int \frac{1}{x+2}dx + \frac{3}{4}\int \frac{1}{x-2}dx$

$= x - \frac{3}{4}\ln|x+2| + \frac{3}{4}\ln|x-2| + c$

$= x + \frac{3}{4}\ln\left|\frac{x-2}{x+2}\right| + c$

Alternative method: $\int \frac{x^2-1}{x^2-4}dx = \int \frac{x^2-4+3}{x^2-4}dx$

$= \int \left(\frac{x^2-4}{x^2-4} + \frac{3}{x^2-4}\right)dx$

$= \int \left(1 + \frac{3}{x^2-4}\right)dx$

$= \int 1dx + 3\int \frac{1}{x^2-2^2}dx$

$= x + 3 \times \frac{1}{2\times 2}\ln\left|\frac{x-2}{x+2}\right| + c$

$= x + \frac{3}{4}\ln\left|\frac{x-2}{x+2}\right| + c$

3(i)Integrate: $\int \frac{dx}{x^2(x-1)}$; [DB. 14;BB. 05,10; RB. 02; CB. 02]

Solution: ধরি, $\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$

$\Rightarrow 1 = Ax(x-1) + B(x-1) + Cx^2$ ……… (i)

$\Rightarrow 1 = Ax^2 - Ax + Bx - B + Cx^2$

$\Rightarrow 1 = (A+C)x^2 + (B-A)x - B$ …………. (ii)

$x = 0$ হলে (i) নং থেকে পাই,, $B = -1$

$x = 1$ হলে (i) নং থেকে পাই,, $C = 1$

এখন (ii) নং থেকে, x^2 সহগ সমীকৃত করে পাই,

$A + C = 0 \Rightarrow A = -C \therefore A = -1$

$\therefore \int \frac{dx}{x^2(x-1)} = -\int \frac{dx}{x} - \int \frac{dx}{x^2} + \int \frac{dx}{x-1}$

$= -\ln|x| + \frac{1}{x} + \ln|x-1| + c$

$= \ln\left|\frac{x-1}{x}\right| + \frac{1}{x} + c$

4(i)Integrate: $\int \frac{xdx}{(x-1)(x^2+1)}$ [DjR. 14; JR.13: CB. 04, 11: DB, 13,08; BB. 01,07; RB. 13]

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Solution: ধরি, $\frac{x}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$

$\Rightarrow x=A(x^2+1)+(Bx+C)(x-1)\ldots\ldots(i)$

$\Rightarrow x=Ax^2+A+Bx^2-Bx+Cx-C$

$\Rightarrow x=(A+B)x^2+(-B+C)x+(A-C)+\cdots(ii)$

$x=1$ হলে (i) নং থেকে পাই, $1=2A\therefore A=\frac{1}{2}$

(ii) নং থেকে, x^2 সহগ সমীকৃত করে পাই,

$A+B=0\therefore B=-A=-\frac{1}{2}$

পুনরায়(ii) নং থেকে ধ্রুবক পদ সমীকৃত করে পাই,

$A-C=0\therefore C=A=\frac{1}{2}$

$\therefore\int\frac{xdx}{(x-1)(x^2+1)}$

$=\frac{1}{2}\int\frac{dx}{x-1}+\frac{1}{2}\int\frac{1-x}{x^2+1}dx$

$=\frac{1}{2}\ln|x-1|-\frac{1}{2}\int\frac{x-1}{x^2+1}dx$

$=\frac{1}{2}\ln|x-1|-\frac{1}{2}\int\frac{x}{x^2+1}dx+\frac{1}{2}\int\frac{dx}{1+x^2}$

$=\frac{1}{2}\ln|x-1|-\frac{1}{2\times2}\int\frac{2x}{x^2+1}dx+\frac{1}{2}\int\frac{dx}{1+x^2}$

$=\frac{1}{2}\ln|x-1|-\frac{1}{4}\ln[1+x^2]+\frac{1}{2}\tan^{-1}x+c$

4(ii) Integrate: $\int\frac{x+2}{(1-x)(x^2+4)}dx$ [JB. 14]

Solution: ধরি, $\frac{x+2}{(1-x)(x^2+4)}\equiv\frac{A}{1-x}+\frac{Bx+C}{x^2+4}$

$\Rightarrow x+2=A(x^2+4)+(Bx+C)(1-x)\ldots\ldots(i)$

$\Rightarrow x+2=Ax^2+4A+Bx-Bx^2+C-Cx$

$\Rightarrow x+2\equiv(A-B)x^2+(B-C)x+4A+C\ldots.(ii)$

$x=1$ হলে (i) নং থেকে পাই,

$3=5A\therefore A=\frac{3}{5}$

(ii) নং থেকে x^2 এর সহগ সমীকৃত করে পাই,

$A-B=0\Rightarrow B=A\ \therefore B=\frac{3}{5}$

এবং $4A+C=2\Rightarrow C=2-\frac{12}{5}\therefore C=-\frac{2}{5}$

$\therefore\int\frac{x+2}{(1-x)(x^2+4)}dx$

$=\frac{3}{5}\int\frac{dx}{1-x}+\frac{3}{5}\int\frac{xdx}{x^2+4}-\frac{2}{5}\int\frac{dx}{x^2+4}$

$=\frac{3}{5}\int\frac{dx}{1-x}+\frac{3}{10}\int\frac{2xdx}{x^2+4}-\frac{2}{5}\int\frac{dx}{x^2+4}$

$=-\frac{3}{5}\ln|1-x|+\frac{3}{10}\ln|x^2+4|-\frac{2}{5}.\frac{1}{2}\tan^{-1}\frac{x}{2}$

$=-\frac{3}{5}\ln|1-x|+\frac{3}{10}\ln|x^2+4|-\frac{1}{5}\tan^{-1}\frac{x}{2}$

Exercise-10.5

1(i) Integrate: $\int\ln xdx$
[CtgB, 04, 08; CB, 06; BB, 04]

Solution: $\int\ln xdx=\int\ln x.1dx$

$=\ln x\int1.dx-\int\left\{\frac{d}{dx}(\ln x)\int1.dx\right\}dx$

$=x\ln x-\int\frac{1}{x},xdx=x\ln x-\int1dx$

$=x\ln x-x+c$

1(ii) Integrate: $\int x\sec^2xdx$ Ctg B.14]

Solution: $\int x\sec^2xdx$

$=x\int\sec^2xdx-\int\left\{\frac{d}{dx}(x)\int\sec^2xdx\right\}dx$

$=x\tan x-\int\tan xdx$

$=x\tan x-\ln|\sec x|+c$

1(iii) Integrate: $\int\frac{xdx}{\sin^2x}$.

Solution: $\int\frac{xdx}{\sin^2x}=\int x\operatorname{cosec}^2xdx$

$=x\int\operatorname{cosec}^2xdx-\int\left\{\frac{d}{dx}(x)\int\operatorname{cosec}^2xdx\right\}dx$

$=-x\cot x-\int1\cdot(-\cot x)dx$

$=-x\cot x+\int\cot xdx$

$=-x\cot x+\ln|\sin x|+c$

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2(i) Integrate: $\int \frac{\ln(\sec^{-1}x)}{x\sqrt{x^2-1}}dx$ **[SB. I4; DB. 08]**

Solution: $\int \frac{\ln(\sec^{-1}x)}{x\sqrt{x^2-1}}dx$

ধরি, $\sec^{-1}x = z$

$\therefore \frac{1}{x\sqrt{x^2-1}}dx = dz$

$= \int \ln z\, dz$

$= \ln z \int dz - \int \left[\frac{d}{dx}(\ln x)\int dz\right]dz$

$= \ln z z - \int \frac{1}{z} zdz = z\ln z - \int dz$

$= z\ln z - z + c$

$= (\sec^{-1} x)\ln(\sec^{-1} x) - (\sec^{-1} x) + c$

2 (ii)Integratet $\int x \ln x\, dx$

Solation: $\int x\ln x dx = \int \ln x \cdot xdx$

$= \ln x \int xdx - \int \left[\frac{d}{dx}(\ln x)\int xdx\right]dx$

$= \frac{x^2}{2}\ln x - \int \frac{1}{x}\frac{x^2}{2}dx$

$= \frac{x^2}{2}\ln x - \int \frac{x}{2}dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int xdx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\frac{x^2}{2} + c = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$

2(iii) Integrate: $\int x\sin^2 xdx.$ **[KUET, 05-06]**

Solution: $\int x\sin^2 xdx = \frac{1}{2}\int x \cdot 2\sin^2 xdx$

$= \frac{1}{2}\int x(1-\cos 2x)dx = \frac{1}{2}\int (x - x\cos 2x)dx$

$= \frac{1}{2}\int x\,dx - \frac{1}{2}\int x\cos 2x)\,dx$

$= \frac{1}{2}\frac{x^2}{2} - \frac{1}{2}\Big[x\int \cos 2xdx$

$- \int \left(\frac{d}{dx}(x)\int cos2x\,dx\right\}dx\Big]$

$= \frac{x^2}{4} - \frac{1}{2}\left[x\frac{\sin 2x}{2} - \int \left\{\frac{\sin 2x}{2}\right\}dx\right]$

$= \frac{x^2}{4} - \frac{1}{2}\left[\frac{x\sin 2x}{2} - \frac{1}{2}\int \sin 2xdx\right]$

$= \frac{x^2}{4} - \frac{1}{2}\left[\frac{x\sin 2x}{2} - \frac{1}{2}\left(\frac{-\cos 2x}{2}\right)\right] + c$

$= \frac{x^2}{4} - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c$

2(iv) Integrate: $\int x\tan^2 xdx$

[RB, 05; SB. 05; DB. 02]

Solution: $\int x\tan^2 x\,dx = \int x(\sec^2 x - 1)dx$

$= \int x\sec^2 xdx - \int xdx$

$= x\int \sec^2 xdx - \int \left\{\frac{d}{dx}(x)\int \sec^2 x\,dx\right\}dx - \frac{x^2}{2}$

$= x\tan x - \int \tan xdx - \frac{x^2}{2}$

$= x\tan x - \ln|\sec x| - \frac{1}{2}x^2 + c$

$= x\tan x - \ln\left|\frac{1}{\cos x}\right| - \frac{x^2}{2} + c$

$= x\tan x + \ln|\cos x| - \frac{x^2}{2} + c$

3(i) Integrate: $\int x^2\cos xdx$

Solution: $\int x^2\cos xdx$

$= x^2\int \cos xdx - \int \left\{\frac{d}{dx}(x^2)\int \cos xdx\right\}dx$

$= x^2\sin x - 2\int x\sin x\,dx$

$= x^2\sin x - 2\Big[x\int \sin xdx$

$- \int \left\{\frac{d}{dx}(x)\int \sin xdx\right\}dx\Big]$

$= x^2\sin x - 2[-x\cos x + \int \cos xdx]$

$= x^2\sin x + 2x\cos x - 2\sin x + c$

3(ii) Integrate: $\int (\ln x)^2 dx$ **[DB12,14]**

Solution: $\int (\ln x)^2 dx = \int (\ln x)^2 \cdot 1dx$

$= (\ln x)^2\int 1dx - \int \left\{\frac{d}{dx}(\ln x)^2\int 1dx\right\}dx$

$= x(\ln x)^2 - \int \frac{2.\ln x}{x}xdx$

$= x(\ln x)^2 - 2\int \ln xdx$

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$$= x(\ln\ x)^2 - 2\left[\ln\ x\int dx - \int\left\{\frac{d}{dx}(\ln\ x)\int dx\right\}dx\right]$$

$$= x(\ln\ x)^2 - 2\left[x\ln\ x - \int dx\right]$$

$$= x(\ln\ x)^2 - 2x\ln\ x + 2x + c$$

4(i) Integrate: $\int \mathbf{e^x sin\ \ 2x\ dx}$
[SB. 10; RB. 04,09; DjB. 09; DB, 03]

Solution: ধরি, $I = \int \mathbf{e^x sin\ \ 2x\ dx}$

$$= e^x\int \sin 2xdx - \int\left\{\frac{d}{dx}(e^x)\int \sin 2xdx\right\}dx$$

$$= -\frac{1}{2}e^x\cos 2x + \frac{1}{2}\int e^x\cos 2xdx$$

$$= -\frac{1}{2}e^x\cos 2x$$

$$+\frac{1}{2}\left[e^x\int \cos 2xdx - \int\left\{\frac{d}{dx}(e^x)\int \cos 2xdx\right\}dx\right]$$

$$= -\frac{1}{2}e^x\cos 2x$$

$$+\frac{1}{2}\left[e^x\cdot\frac{\sin\ 2x}{2} - \int e^x\cdot\frac{\sin\ 2x}{2}dx\right]$$

$$\Rightarrow \mathrm{I} = -\frac{1}{2}e^x\cos\ 2x + \frac{1}{4}e^x\sin\ 2x - \frac{1}{4}\int e^x\sin\ 2xdx$$

$$\Rightarrow \mathrm{I} = \frac{1}{4}e^x\sin 2x - \frac{1}{2}e^x\cos 2x - \frac{1}{4}I$$

$$\Rightarrow \frac{5}{4}I = \frac{1}{4}e^{\mathrm{x}}(\sin\ 2x - 2\cos\ 2x)$$

$$\mathrm{I} = \frac{1}{5}e^x(\sin\ 2x - 2\cos\ 2x) + c$$

$$\therefore \int e^x\sin\ 2xdx = \frac{1}{5}e^x(\sin\ 2x - 2\cos\ 2x) + c$$

4(ii) Integratet $\int \boldsymbol{x\sin^{-1}\ x dx}$
[DB, 07; MB, 02, 03, 06]

Solution: $\int x\sin^{-1}\ xdx$

ধরি, $I = \sin^{-1}\ x\int xdx - \int\left\{\frac{d}{dx}(\sin^{-1}\ x)\int xdx\right\}dx$

$$I = \frac{1}{2}x^2\sin^{-1}\ x - \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx$$

$$\mathrm{I} = \frac{1}{2}x^2\sin^{-1}\ x - \mathrm{I}_1$$

where $I_1 = \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}dx$

$$= \frac{1}{2}\int\frac{\sin^2\theta\,.\cos\ \theta d\theta}{\cos\ \theta}$$

> For second part:
>
> ধরি, $x = \sin\ \theta$
>
> $\therefore dx = \cos\ \theta d\theta$

$$= \frac{1}{4}\int 2\sin^2\ \theta = \frac{1}{4}\int(1-\cos\ 2\theta)d\theta$$

$$= \frac{1}{4}\left[\theta - \frac{\sin\ 2\theta}{2}\right] = \frac{1}{4}\theta - \frac{1}{8}\cdot 2\sin\ \theta\cdot\cos\ \theta$$

$$= \frac{1}{4}\theta - \frac{1}{4}\sin\ \theta\cdot\sqrt{\cos^2\ \theta}$$

$$= \frac{1}{4}\theta - \frac{1}{4}\sin\ \theta\cdot\sqrt{1-\sin^2\ \theta}$$

$$= \frac{1}{4}\sin^{-1}\ x - \frac{1}{4}x\sqrt{1-x^2}$$

$$\therefore \mathrm{I} = \frac{1}{2}x^2\sin^{-1}\ x - \frac{1}{4}\sin^{-1}\ x + \frac{1}{4}x\sqrt{1-x^2} + c$$

4(iii) Integrate: $\int \boldsymbol{x\sin^{-1}\ x^2 dx}$
[RB. 06; DB. 05; RB.13]

Solution: $\int x\sin^{-1}\ x^2dx$ ধরি, $x^2 = z$

$= \int\sin^{-1}z\cdot\frac{dz}{2}$ $\therefore 2xdx = dx$

$$= \frac{1}{2}\int\sin^{-1}\ zdz$$

$$= \frac{1}{2}\left[\sin^{-1}\ z\int dz - \int\left\{\frac{d}{dz}(\sin^{-1}\ z)\int dz\right\}dz\right]$$

$$= \frac{1}{2}\left[z\sin^{-1}\ z - \int\frac{z}{\sqrt{1-z^2}}dz\right]$$

$$= \frac{1}{2}\left[z\sin^{-1}\ z + \frac{1}{2}\int\frac{-2z}{\sqrt{1-z^2}dz}\right]$$

$$= \frac{1}{2}\left[z\sin^{-1}\ z + \frac{1}{2}\cdot 2\sqrt{1-z^2}\right] + c$$

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[Formula: $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$]

$= \frac{1}{2}\left[z\sin^{-1} z + \sqrt{1-z^2}\right] + c$

$= \frac{1}{2}\left[x^2\sin^{-1}(x^2) + \sqrt{1-x^4}\right] + c$

5(i)Integrate: $\int \tan^{-1} x dx$

[JB. 13; DjB. 12; DB.04; CB. 02]

Solution: $\int \tan^{-1} x dx = \int \tan^{-1} x . 1dx$

$= \tan^{-1} x \int 1dx - \int \left\{\frac{d}{dx}(\tan^{-1} x)\int 1dx\right\}dx$

$= x\tan^{-1} x - \int \frac{x}{1+x^2}dx$

$= x\tan^{-1} x - \frac{1}{2}\int \frac{2x}{1+x^2}dx$

$= x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| + c$

5(ii) Integrate: $\int \sin^{-1} x dx$

[DB.14; BB.12; JB. 10; SB, 03]

Solution: $\int \sin^{-1} x\, dx = \int \sin^{-1} x . 1dx$

$= \sin^{-1} x \int 1\,dx - \int \left\{\frac{d}{dx}(\sin^{-1} x)\int 1dx\right\}dx$

$= x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}dx$

$= x\sin^{-1} x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}dx$

$= x\sin^{-1} x + \frac{1}{2}.2\sqrt{1-x^2} + c$

[Formula: $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$]

$= x\sin^{-1} x + \sqrt{1-x^2} + c$

5(iii) Integrate: $\int \cos^{-1} x\, dx$

[CB. 14; CtgB. 07,12; SB.]

Solution: $\int \cos^{-1} x dx = \int \cos^{-1} x . 1dx$

$= x\cos^{-1} x + \int \frac{xdx}{\sqrt{1-x^2}}$

$= x\cos^{-1} x - \frac{1}{2}\int \frac{-2xdx}{\sqrt{1-x^2}}$

$= x\cos^{-1} x - \frac{1}{2}\cdot 2\sqrt{1-x^2} + c$

[Formula: $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$]

$= x\cos^{-1} x - \sqrt{1-x^2} + c$

6(i)Integrate; $\int \frac{e^x}{x}(1 + x\ln x)dx$

|JB. 07; BB, 01; DJB. 13]

Solution: $\int \frac{e^x}{x}(1 + x\ln x)dx$

$= \int e^x\left(\frac{1}{x} + \ln x\right)dx$ [Let, $f(x) = \ln x : f'(x) = \frac{1}{x}$]

$= e^x \ln x + c$

[Formula: $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$]

6(ii) Integrate: $\int e^x \sec x\,(1 + \tan x)dx$

[JB.11; RB, 03; BUET, 04 -05]

Solution: $\int e^x \sec x\,(1 + \tan x)dx$

$= \int e^x(\sec x + \sec x\tan x)dx$

[ধরি, $f(x) = \sec x \therefore f'(x) = \sec x\tan x$]

$= e^x \sec x + c$

[Formula: $\int e^x(f(x) + f'(x)\}dx = e^x f(x) + c$]

6(iii)Integrate: $\int e^x\{\tan x - \ln(\cos x)\}dx$

solutiont $\int e^x(\tan x - \ln(\cos x))dx$

[ধরি, $f(x) = -\ln(\cos x)$

$\therefore f'(x) = -\frac{1}{\cos x}(-\sin x) = \tan x$]

$= e^x \cdot \{-\ln(\cos x)\} + c$

$= -e^x \ln(\cos x) + c$

[Formula: $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + c$]

6(iv) Integrate: $\int e^x\left\{\frac{1}{1-x} + \frac{1}{(1-x)^2}\right\}dx$

Solution: $\int e^x\left\{\frac{1}{1-x} + \frac{1}{(1-x)^2}\right\}dx$

ধরি, $f(x) = \frac{1}{1-x} \therefore f'(x) = \frac{-1}{(1-x)^2}(-1) = \frac{1}{(1-x)^2}$]

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$$e^x \cdot \frac{1}{1-x} + c = \frac{e^x}{1-x} + c$$
[Formula: $\int e^x\{f(x)+f'(x)\}dx = e^x f(x) + c$]

6(v) Integrate: $\int \frac{xe^x}{(x+1)^2}dx$[JB, 09,12; DB. 11:RB. 12; CtgB. 13; BUET, 06-07; DU.11]

Solution: $\int \frac{xe^x}{(x+1)^2}dx = \int \frac{(x+1-1)e^x}{(x+1)^2}dx$

$= \int_{e^x} \left\{\frac{x+1}{(x+1)^2} - \frac{1}{(x+1)^2}\right\}dx$

$= \int e^x \left\{\frac{1}{x+1} + \frac{-1}{(x+1)^2}\right\}dx$

[Let, $f(x) = \frac{1}{x+1} \therefore f'(x) = \frac{-1}{(x+1)^2}$]

$= \frac{e^x}{x+1} + c$

[Formula: $\int e^x\{f(x)+f'(x)\}dx = e^x f(x) + c$]

6(vi) Integrate: $\int \frac{e^x(x^2+1)}{(x+1)^2}dx$ [BUET. 03]

Solution: $\int \frac{e^x(x^2+1)}{(x+1)^2}dx$

$= \int \frac{e^x\{(x+1)^2 - 2x\}}{(x+1)^2}dx$

$= \int e^x dx - \int \frac{2xe^x dx}{(x+1)^2}$

$= \int e^x dx - 2\int \frac{e^x(x+1-1)dx}{(x+1)^2}$

$= \int e^x dx - 2\int e^x \left\{\frac{x+1}{(x+1)^2} - \frac{1}{(x+1)^2}\right\}dx$

$= \int e^x dx - 2\int e^x \left\{\frac{1}{x+1} + \frac{-1}{(x+1)^2}\right\}dx$

[ধরি, $f(x) = \frac{1}{x+1} \therefore f'(x) = \frac{-1}{(x+1)^2}$]

$= e^x - 2e^x \cdot \frac{1}{x+1} + c$

$= e^x\left(1 - \frac{2}{x+1}\right) + c = \frac{e^x(x-1)}{x+1} + c$

6(vii) Integrate: $\int e^{5x}\left(5\ln x + \frac{1}{x}\right)dx$ [CtgB. 09]

Solution: $\int e^{5x}\left(5\ln x + \frac{1}{x}\right)dx$

[ধরি, $f(x) = \ln x \therefore f'(x) = \frac{1}{x}$]

$= e^{5x}\ln x + c$

[Formula: $\int e^{ax}(af(x)+f'(x)\}dx = e^{ax}f(x) + c$]

Exercise-10.6

1.i)Evaluate: $\int_0^2 5x^4 dx$

Solution: $\int_0^2 5x^4 dx = 5\int_0^2 x^4 dx$

$= 5.\left[\frac{x^5}{5}\right]_0^2 = 5 \cdot \left(\frac{2^5}{5} - 0\right) = 32$

(ii) Evaluate: $\int_0^3 (3 - 2x + x^2)dx$ [CB. 06, 07; BB, 08]

Solution:$\int_0^3 (3 - 2x + x^2)dx$

$= \left[3x - 2\frac{x^2}{2} + \frac{x^3}{3}\right]_0^3 = 9$

(iii)Evaluate:$\int_{-1}^{-2}(2 + 3y + 5y^2)\,dy$

Solution:$\int_{-1}^{-2}(2 + 3y + 5y^2)\,dy$

$= \left[2y + 3\frac{y^2}{2} + 5\frac{y^3}{3}\right]_{-1}^{-2}$

$= \left\{2(-2) + \frac{3}{2}(-2)^2 + \frac{5}{3}(-2)^3\right\}$

$-\left\{2(-1) + \frac{3}{2}(-1)^2 + \frac{5}{3}(-1)^2\right\}$

$= \left(-4 + 6 - \frac{5\times 8}{3}\right) - \left(-2 + \frac{3}{2} - \frac{5}{3}\right)$

$= 2 - \frac{40}{3} + 2 - \frac{3}{2} + \frac{5}{3} = \frac{12 - 80 + 12 - 9 + 10}{6}$

$= -\frac{55}{6}$

2.(i) Evaluate: $\int_{-\pi/2}^{\pi/2} \frac{\sec x + 1}{\sec x}dx$ |CB, 09; JB. 03, 06, 13 |

Solution: $\int_{-\pi/2}^{\pi/2}\frac{\sec x+1}{\sec x}dx$

$=\int_{-\pi/2}^{\pi/2}(1+\cos x)dx=[x+\sin x]_{-n/2}^{\pi/2}$

$=\left(\frac{\pi}{2}+\sin\frac{\pi}{2}\right)-\left\{-\frac{\pi}{2}+\sin\left(-\frac{\pi}{2}\right)\right\}$

$=\frac{\pi}{2}+1+\frac{\pi}{2}+1=\pi+2$

(ii) Evaluate: $\int_{\pi/2}^{\pi}(1+\sin 2\theta)d\theta$ [MB. 01]

Solution: $\int_{\pi/2}^{\pi}(1+\sin 2\theta)d\theta$

$=\left[\theta-\frac{1}{2}\cos 2\theta\right]_{\pi/2}^{\pi}$

$=\left(\pi-\frac{1}{2}\cos 2\pi\right)-\left(\frac{\pi}{2}-\frac{1}{2}\cos\pi\right)$

$=\left(\pi-\frac{1}{2}\right)-\left(\frac{\pi}{2}+\frac{1}{2}\right)=\frac{\pi}{2}-1$

(iii) Evaluater $\int_0^{\frac{\pi}{2}}\sqrt{1+\sin\theta}\,d\theta$ [BB.11]

Solution: $\sqrt{1+\sin\theta}$

$=\sqrt{\left(\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)}$

$=\sqrt{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2}=\sin\frac{\theta}{2}+\cos\frac{\theta}{2}$

$\therefore\int_0^{\frac{\pi}{2}}\sqrt{1+\sin\theta}\,d\theta$

$=\int_0^{\frac{\pi}{2}}\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)d\theta$

$=\left[\frac{-\cos\frac{\theta}{2}}{\frac{1}{2}}+\frac{\sin\frac{\theta}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}}=2\left[\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right]_0^{\frac{\pi}{2}}$

$=2\left\{\left(\sin\frac{\pi}{4}-\cos\frac{\pi}{4}\right)-(\sin 0-\cos 0)\right\}$

$=2\left\{\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)-(0-1)\right\}=2.1=2$

3.(i) Evaluate: $\int_0^{\pi/2}\cos^2 xdx$ [SB. 11; RB. 05, 09; CtgB. 04; DB. 02]

Solution: $\int_0^{\pi/2}\cos^2 xdx$

$=\frac{1}{2}\int_0^{\pi/2}2\cos^2 xdx=\frac{1}{2}\int_0^{\pi/2}(1+\cos 2x)$

$=\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]_0^{\pi/2}$

$=\frac{1}{2}\left(\frac{\pi}{2}+\frac{1}{2}\sin\pi\right)-\frac{1}{2}\left(0+\frac{1}{2}\sin 0\right)$

$=\frac{\pi}{4}+\frac{1}{4}.0=\frac{\pi}{4}$

(ii) Evaluate: $\int_0^{\pi/2}\cos^3 xdx$
[SB. 06, 12; JB. 07, 09, 13; DjB. 13]

Solution: $\int_0^{\pi/2}\cos^3 xdx=\frac{1}{4}\int_0^{\pi/2}4\cos^3 x\,dx$

$=\frac{1}{4}\int_0^{\pi/2}(\cos 3x+3\cos x)dx$

$=\frac{1}{4}\left[\frac{1}{3}\sin 3x+3\sin x\right]_0^{\pi/2}$

$=\frac{1}{4}\left(\frac{1}{3}\sin\frac{3\pi}{2}+3\sin\frac{\pi}{2}\right)-0$

$=\frac{1}{4}\left(\frac{1}{3}(-1)+3.1\right)=\frac{1}{4}\left(\frac{-1+9}{3}\right)=\frac{1}{4}\frac{8}{3}=\frac{2}{3}$

(iii) Evaluate: $\int_0^{\pi/2}\sin^3 xdx$

Solution: $\int_0^{\pi/2}\sin^3 xdx$

$=\frac{1}{4}\int_0^{\pi/2}(3\sin x-\sin 3x)dx$

$=\frac{1}{4}\left[-3\cos x+\frac{1}{3}\cos 3x\right]_0^{\pi/2}$

$=\frac{1}{4}\left\{\left(-3\cos\frac{\pi}{2}+\frac{1}{3}\cos\frac{3\pi}{2}\right)-\left(-3\cos 0+\frac{1}{3}\cos 0\right)\right\}$

$=0-\frac{1}{4}\left(-3+\frac{1}{3}\right)=\frac{2}{3}$

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(iv) Evaluate: $\int_0^{\pi/2} \cos^4 x dx$;

Solution: $\cos^4 x = (\cos^2 x)^2 = \frac{1}{4}(2\cos^2 x)^2$

$= \frac{1}{4}(1+\cos 2x)^2$

$= \frac{1}{4}(1+2\cos 2x+\cos^2 2x)$

$= \frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{4}\cos^2 2x$

$= \frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{8}.2\cos^2 2x$

$= \frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{8}(1+\cos 4x)$

$= \frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{8}+\frac{1}{8}\cos 4x$

$= \frac{3}{8}+\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x$

$\therefore \int_0^{\pi/2} \cos^4 x dx$

$= \int_0^{\pi/2}\left(\frac{3}{8}+\frac{1}{2}\cos 2x+\frac{1}{8}\cos 4x\right)dx$

$= \left[\frac{3}{8}x+\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x\right]_0^{\frac{\pi}{2}}$

$= \left(\frac{3}{8}.\frac{\pi}{2}+\frac{1}{4}\sin\pi+\frac{1}{32}\sin 2\pi\right)-0$

$= \frac{3\pi}{16}+0+0 = \frac{3\pi}{16}$

4.(i)Evaluate: $\int_0^{\pi/2}\cos 2x \cos 3x dx$
|DB. 14; CB. 00; CtgB. 03|

Solution: $\int_0^{\pi/2}\cos 2x\cdot\cos 3x dx$

$= \frac{1}{2}\int_0^{\pi/2} 2\cos 3x\cdot\cos 2x dx$

$= \frac{1}{2}\int_0^{\pi/2}\{\cos(3x+2x)+\cos(3x-2x)\}dx$

$= \frac{1}{2}\int_0^{\pi/2}(\cos 5x+\cos x)dx$

$= \frac{1}{2}\left[\frac{1}{5}\sin 5x+\sin x\right]_0^{\pi/2}$

$= \frac{1}{2}\left(\frac{1}{5}\sin\frac{5\pi}{2}+\sin\frac{\pi}{2}\right)-0 = \frac{1}{2}\left(\frac{1}{5}+1\right) = \frac{3}{5}$

(ii) Evaluate: $\int_0^{\pi/2}\sin x\sin 2x dx$ [RB. 08: JB. 01,08; CtgB. 02, 06; DjB. 13]

Solution: $\sin x\cdot\sin 2x = \frac{1}{2}2\sin 2x\sin x$

$= \frac{1}{2}(\cos(2x-x)-\cos(2x+x))$

$= \frac{1}{2}(\cos x-\cos 3x)$

$\therefore \int_0^{\pi/2}\sin x\sin 2x dx$

$= \int_0^{\pi/2}\frac{1}{2}(\cos x-\cos 3x)dx$

$= \frac{1}{2}\left[\sin x-\frac{1}{3}\sin 3x\right]_0^{\pi/2}$

$= \frac{1}{2}\left(\sin\frac{\pi}{2}-\frac{1}{3}\sin\frac{3\pi}{2}\right)-0 = \frac{1}{2}\left(1+\frac{1}{3}\right)$

$= \frac{1}{2}\times\frac{4}{3} = \frac{2}{3}$

(iii) Evaluate: $\int_0^{\pi/2}\sin^2 x\sin 3x dx$
[BB, 05; SB. 03; RB. 00; MB. 04]

Solution:

$\sin^2 x\cdot\sin 3x\, dx = \frac{1}{2}2\sin^2 x\cdot\sin 3x$

$= \frac{1}{2}(1-\cos 2x)\sin 3x$

$= \frac{1}{2}\sin 3x-\frac{1}{2}\sin 3x\cdot\cos 2x$

$= \frac{1}{2}\sin 3x-\frac{1}{4}2\sin 3x\cos 2x$

$= \frac{1}{2}\sin 3x-\frac{1}{4}\{\sin(3x+2x)+\sin(3x-2x)\}$

$= \frac{1}{2}\sin 3x-\frac{1}{4}(\sin 5x+\sin x)$

$= \frac{1}{2}\sin 3x-\frac{1}{4}\sin 5x-\frac{1}{4}\sin x$

$\therefore \int_0^{\pi/2}\sin^2 x\cdot\sin 3x dx$

$= \int_0^{\pi/2}\left(\frac{1}{2}\sin 3x-\frac{1}{4}\sin 5x-\frac{1}{4}\sin x\right)dx$

$= \left[-\frac{1}{6}\cos 3x+\frac{1}{20}\cos 5x+\frac{1}{4}\cos x\right]_0^{\pi/2}$

$$= 0 - \left(-\frac{1}{6} + \frac{1}{20} + \frac{1}{4}\right) = -\frac{2}{15}$$

5.(i)Evaluate: $\int_0^{\pi/2} \frac{dx}{1+\cos x}$
[DB. 11; SB. 11; BB, 08; RB. 01]

Solation: $\int_0^{\pi/2} \frac{dx}{1+\cos x} = \int_0^{\pi/2} \frac{1}{2\cos^2 \frac{x}{2}} dx$

$$= \frac{1}{2}\int_0^{\pi/2} \sec^2 \frac{x}{2} dx = \frac{1}{2}\left[\frac{\tan \frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi/2}$$

$$= \left[\tan \frac{x}{2}\right]_0^{\pi/2} = \tan \frac{\pi}{4} - \tan 0 = 1 - 0 = 1$$

(ii) Evaluate: $\int_0^{\pi/4} \frac{dx}{1+\sin x}$
[CB. 14; SB. 14; DB. 12; BB. 12, 14; RB. 10; JB. 08; DjR. 14, 10; BUET. 05-06]

Solution: $\int_0^{\pi/4} \frac{dx}{1+\sin x} = \int_0^{\pi/4} \frac{(1-\sin x)}{1-\sin^2 x} dx$

$$= \int_0^{\pi/4} \frac{1-\sin x}{\cos^2 x} dx$$

$$= \int_0^{\pi/4} \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right) dx$$

$$= \int_0^{\pi/4} (\sec^2 x - \sec x \tan x) dx$$

$$= [\tan x - \sec x]_0^{\pi/4}$$

$$= \left(\tan \frac{\pi}{4} - \sec \frac{\pi}{4}\right) - (\tan 0 - sec 0)$$

$$= (1 - \sqrt{2}) + 1 = 2 - \sqrt{2}$$

(iii) Evaluate: $\int_0^{\pi/3} \frac{dx}{1-\sin x}$
[SB. 10;DB. 01, 08, 09, 13; JB. 09; CtgB. 01; RB. 13]

Solution: $\int_0^{\pi/3} \frac{1}{1-\sin x} dx = \int_0^{\pi/3} \frac{1+\sin x}{1-\sin^2 x} dx$

$$= \int_0^{\pi/3} \frac{1+\sin x}{\cos^2 x} dx$$

$$= \int_n^{\pi/3} \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right) dx$$

$$= \int_0^{\pi/3} (\sec^2 x + \sec x \tan x) dx$$

$$= [\tan x + \sec x]_0^{\pi/3}$$

$$= \left(\tan \frac{\pi}{3} + \sec \frac{\pi}{3}\right) - \sec 0 = (\sqrt{3} + 2) - 1$$

$$= \sqrt{3} + 1$$

6.(i) Evaluate: $\int_0^{\pi/4} \tan^2 x \sec^2 x dx$ **[CB, 04, 06 [DB. 03, 05, 13; CtgB, 04, 11}**

Solution: $\int_0^{\pi/4} \tan^2 x \sec^2 x \, dx$

$= \int_0^1 z^2 dz$ | ধরি, $\tan x - z$; $\therefore \sec^2 xdx = dz$

$= \frac{1}{3}[z^3]_0^1$ | যখন, $x = 0$ তখন, $z = 0$

$= \frac{1}{3}$ | যখন, $x = \frac{\pi}{4}$ তখন, z=1

(ii) Evaluate: $\int_0^{\pi/4} 4\tan^3 x \sec^2 x dx$
[DB. 11; BB. 11; CB. 09; SB. 13}

Solution: $\int_0^{\pi/4} 4 \tan^3 x \sec^2 xdx$

$= 4\int_0^1 z^3 dz = 4\frac{1}{4}[z^4]_0^1$

$= 1 - 0$

$= 1$

ধরি, $\tan x = z$
$\therefore \sec^2 xdx = dz$
যখন, $x = 0$ তখন, $z = 0$
যখন, $x = \frac{\pi}{4}$ তখন, $z = 1$

(iii) Evaluate: $\int_0^{\pi/4} \tan^6 x \sec^2 x dx$

Solution:

$\int_0^{\pi/4} \tan^6 x \cdot \sec^2 x \, dx$

$= \int_0^1 z^6 dz$

$= \frac{1}{7}[z^7]_0^1 = \frac{1}{7}$

ধরি, $\tan x = z$
$\therefore \sec^2 xdx = dz$
যখন, $x = 0$ তখন, z = 0
যখন, x = $\frac{\pi}{4}$ তখন, $z = 1$

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(iv) Evaluate: $\int_0^{\pi/4}(\tan^3 x+\tan x)dx$
[CB. 08; JB, 01,05 |

Solution: $\int_0^{\pi/4}(\tan^3 x+\tan x)dx$

$\int_0^{\pi/4}\tan x(\tan^2 x+1)dx$

$=\int_0^{\pi/4}\tan x\sec^2 x\,dx$

ধরি, $\tan x=z$
$\therefore \sec^2 x dx=dz$

$=\int_0^1 z dz=\frac{1}{2}[z^2]_0^1=\frac{1}{2}$

(v) Evaluate: $\int_{\pi/3}^{\pi/2}\frac{\cos^5 x}{\sin^7 x}dx$ **[DB. 12; DjB. 11; CtgB. 08; RB. 07; JB. 05; BUET, 09-10]**

Solution: $\int_{\pi/3}^{\pi/2}\frac{\cos^5 x}{\sin^7 x}dx$

$=\int_{\pi/3}^{\pi/2}\frac{\cos^5 x}{\sin^5 x}\frac{1}{\sin^2 x}dx$

$=\int_{\pi/3}^{\pi/2}\cot^5 x\,\mathrm{cosec}^2 x dx$

ধরি, cotx=z
$-\mathrm{cosec}^2 x dx=dz$
$\mathrm{cosec}^2 x dx=-dz$

যখন, $x=\pi/2$
তখন, $z=0$

যখন, $x=\pi/3$
তখন, $z=\frac{1}{\sqrt{3}}$

$=\int_{\pi/3}^{\pi/2}-z^5 dz$

$=-\left\{0-\frac{1}{6}\left(\frac{1}{\sqrt{3}}\right)^6\right\}=\frac{1}{6}\frac{1}{27}=\frac{1}{162}$

(vi) Evaluate: $\int_0^1\frac{1+x}{1+x^2}dx$ **KCB. 01,02,12; SB. 02, 05, 14; DB. 09; DjB. 11; BB. 07; Ctg B. 11]**

Solution: $\int_0^1\frac{1+x}{1+x^2}dx=\int_0^1\frac{1}{1+x^2}+\int_0^1\frac{x}{1+x^2}dx$

$=\int_0^1\frac{1}{1+x^2}dx+\frac{1}{2}\int_0^1\frac{2x}{1+x^2}dx$

$=[\tan^{-1} x]_0^1+\frac{1}{2}[\ln|1+x^2|]_0^1$

$=(\tan^{-1} 1-0)+\frac{1}{2}(\ln|2|-\ln|1|\}$

$=\frac{\pi}{4}+\frac{1}{2}\ln 2$

(vii) Evaluate: $\int_0^2\frac{dx}{\sqrt{a^2-x^2}}$

Solution: $\int_0^a\frac{dx}{\sqrt{a^2-x^2}}=\left[\sin^{-1}\frac{x}{a}\right]_0^a$

$=\sin^{-1} 1-0=\frac{\pi}{2}$

viii)Evaluate: $\int_0^1\frac{dx}{\sqrt{4-3x^2}}$ **[DB. 03; CB. 01]**

Solution: $\int_0^1\frac{dx}{\sqrt{4-3x^2}}$

$=\frac{1}{\sqrt{3}}\int_0^1\frac{dx}{\sqrt{\left(\frac{2}{\sqrt{3}}\right)^2-x^2}}=\frac{1}{\sqrt{3}}\left[\sin^{-1}\frac{\sqrt{3}x}{2}\right]_0^1$

$=\frac{1}{\sqrt{3}}\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)-0\right]=\frac{1}{\sqrt{3}}\cdot\frac{\pi}{3}=\frac{\pi}{3\sqrt{3}}$

(ix) Evaluate: $\int_0^1\frac{dx}{\sqrt{2x-x^2}}$

[CUET. 11-12; RUET. 12-13; KUET. 12-13}

Solution: $\int_0^1\frac{dx}{\sqrt{2x-x^2}}=\int_0^1\frac{dx}{\sqrt{1-1+2x-x^2}}$

$=[\sin^{-1}(x-1)]_0^1=[\sin^{-1}(x-1)]_0^1$

$=[\sin^{-1} 0-\sin^{-1}(-1)]=\frac{\pi}{2}$

7.(i) Evaluate: $\int_0^1\frac{xdx}{\sqrt{1-x^2}}$

[RB. 12; DB, 07; JB.10]

Solution: $\int_0^1\frac{xdx}{\sqrt{1-x^2}}$

ধরি, $1-x^2=z$
$\therefore -2xdx=dz$
$\Rightarrow xdx=-\frac{dz}{2}$

$= \int_1^0 \frac{\left(-\frac{1}{2}dz\right)}{\sqrt{z}}$ If $x = 0$ then $z = 1$
If $x = 1$ then $z = 0$

$= -\int_1^0 \frac{1}{2\sqrt{z}}dz$

$= -[\sqrt{z}]_1^0 = -(0-1) = 1$

(ii) Evaluate: $\int_0^1 \frac{xdx}{\sqrt{4-x^2}}$

[RB. 10; BB. 10; CB. 05, 10]

Solution: $\int_0^1 \frac{xdx}{\sqrt{4-x^2}}$

$= \int_4^3 \frac{-\frac{1}{2}dz}{\sqrt{z}}$

$= -\int_4^3 \frac{1}{2\sqrt{z}}dz$

$= -[\sqrt{z}]_4^3$

$= -(\sqrt{3}-\sqrt{4})$

$= 2-\sqrt{3}$

Let, $4 - x^2 = z$
$\therefore -2xdx = dz$
$\Rightarrow xdx = -\frac{dz}{2}$
If $x = 0$ then $z = 4$
If $x = 1$ then $z = 3$

(iii) Evaluate: $\int_0^2 \frac{xdx}{\sqrt{9-2x^2}}$**; [BUET, 09-10, DB. 15 CtgB. 14; SB. 14; CB. 12; BB. 10;JB, 02;**

Solution: $\int_0^2 \frac{xdx}{\sqrt{9-2x^2}}$

$= \int_9^1 \frac{\left(-\frac{1}{4}dz\right)}{\sqrt{z}}$

$= -\frac{1}{4} \times 2\int_9^1 \frac{1}{2\sqrt{z}}dz$

$= -\frac{1}{2}[\sqrt{z}]_9^1$

$= -\frac{1}{2}(1-3)$

$= 1$

ধরি, $9 - 2x^2 = z$
$\Rightarrow -4xdx = dz$
$\therefore xdx = -\frac{1}{4}dz$
If $x = 0$ then $z = 9$
If $x = 2$ then $z = 1$

(iv) Evaluate: $\int_0^1 x^3\sqrt{1+3x^4}dx$
[JB. 12; SB, 08, 12; CB, 07, 10; RB, 05, 07, 09; BB, 04, 09]

Solution: $\int_0^1 x^3\sqrt{1+3x^4}dx$

$= \int_1^4 \frac{1}{12}\sqrt{z}dz$

$= \frac{1}{12}\left[\frac{2}{3}z^{\frac{3}{2}}\right]_1^4$

$= \frac{1}{12}\cdot\frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$

$= \frac{1}{18}(2^3 - 1)$

$= \frac{7}{18}$

ধরি, $1 + 3x^4 = z$
$\Rightarrow 12x^3dx = dz$
$\therefore x^3dx = \frac{1}{12}dz$
$x = 0$ হলে, $z = 1$
$x = 1$ হলে, $z = 4$

8(i) Evaluate: $\int_0^{\pi/2} \cos^3 x \sin\ xdx$
[BB. 11; DjB, 10; DB, 03]

$= \int_0^{\pi/2} \cos^3 x \sin xdx$

$= -\frac{1}{6}(0-1) = \frac{1}{6}$

ধরি, cosx= z
$\therefore -\sin x dx = dz$
x=0 হলে, z=1
$x = \frac{\pi}{2}$ হলে, $z = 0$

8**(ii) Evaluate:** $\int_0^{\frac{\pi}{2}} (1+\cos x)^2 \sin xdx$
[CtgB, 11; SB, 02; BUET. 08 – 09]

ধরি, $1 + \cos x = z$
$\Rightarrow -\sin xdx = dz$
x=0 হলে, , z=2
$x = \frac{\pi}{2}$ হলে, $z = 1$

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Solution: $\int_0^{\frac{\pi}{2}} (1+\cos x)^2 \sin x dx$

$= -\int_2^1 z^2 dz = -\frac{1}{3}[z^3]_2^1$

$= -\frac{1}{3}(1-2^3)$

$=\frac{7}{3}$

(iii) Evaluate: $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x} dx$

[RB. 13; SB. 13]

Solution:

$\int_0^{\pi/2} \frac{\cos x dx}{1+\sin^2 x}$

ধরি, $\sin x = z$
$\therefore \cos xdx = dz$
$x = 0$ হলে, $z = 0$
$x = \frac{\pi}{2}$ হলে, $z = 1$

$= [\tan^{-1} z]_0^1$

$= (\tan^{-1} 1 - 0)$

$= \frac{\pi}{4}$

(iv) Evaluate: $\int_0^{\pi/2} \frac{\cos xdx}{9-\sin^2 x}$

[CB. 10; BB, 10; DB, 05; CtgB. 09; DjB. 13]

Solution: $\int_0^{\pi/2} \frac{\cos xdx}{9-\sin^2 x} = \int_0^1 \frac{dz}{9-z^2}$

$= \frac{1}{2.3}\left[\ln \left|\frac{3+z}{3-z}\right|\right]_0^1$

$= \frac{1}{6}\ln|2| - \frac{1}{6}\ln 1$

$= \frac{1}{6}\ln 2$

(v) Evaluate:

$\int_0^{\pi/2} \cos^5 xdx$

[BIT. 95-96]

Solution:

$\int_0^{\pi/2} \cos^5 x\, dx$

ধরি, $\sin x = z$
$\therefore \cos xdx = dz$
$x = 0$ then $z = 0$
$x = \frac{\pi}{2}$ then $z = 1$

$= \int_0^{\pi/2} \cos^4 x \cdot \cos x\, dx$

$= \int_0^{\pi/2} (\cos^2 x)^2 \cos dx$

$= \int_0^{\pi/2} (1-\sin^2 x)^2 \cdot \cos xdx$

ধরি, $\sin x = z$
$\therefore \cos xdx = dz$
$x = 0$ হলে, $z = 0$
$x = \frac{\pi}{2}$ হলে, $z = 1$

$= \int_0^1 (1-z^2)^2 dz$

$= \int_0^1 (1-2z^2+z^4)dz$

$= \left[z - 2\frac{z^3}{3} + \frac{z^5}{5}\right]_0^1$

$= \left(1-\frac{2}{3}+\frac{1}{5}\right) - (0-0+0)$

$= \frac{15-10+3}{15} = \frac{8}{15}$

(vi) Evaluate: $\int_0^{\pi/2} \frac{\cos^3 x}{\sqrt{\sin x}} dx$

[RB.12; CtgB. 10; BB. 10]

Solution: $\int_0^{\pi/2} \frac{\cos^3 x}{\sqrt{\sin x}} dx = \int_0^{\pi/2} \frac{\cos^2 x \cdot \cos x}{\sqrt{\sin x}} dx$

$= \int_0^{\pi/2} \frac{(1-\sin^2 x)\cdot \cos x}{\sqrt{\sin x}} dx$

$= \int_0^1 \frac{(1-z^2)\cdot dz}{\sqrt{z}}$

ধরি, $\sin x = z$
$\therefore$ cosx $dx -$ dx
$x = 0$ হলে, $z = 0$
$x = \frac{\pi}{2}$ হলে, $z = 1$

$= \int_0^1 \left(z^{-\frac{1}{2}} - z^{\frac{1}{2}}\right) dz$

$= \left[2\sqrt{z} - \frac{2}{5}z^{\frac{5}{2}}\right]_0^1$

$= \left(2-\frac{2}{5}\right) - 0 = \frac{8}{5}$

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(vii) Evaluate: $\int_0^{\pi/2} \cos^3 x\sqrt{\sin x}dx$

[RB. 14; DB. , 06, 14; JB02,12;CB11;. SB, 08,11, 13; BB,09, 14; BUET. 10 − 11]

Solution: $\int_0^{\pi/2} \cos^3 x\sqrt{\sin x}dx$

$= \int_0^{\pi/2} \cos^2 x\sqrt{\sin x} \cdot \cos xdx$

$= \int_0^{\pi/2} (1-\sin^2 x)\sqrt{\sin x} \cdot \cos xdx$

$= \int_0^1 (1-z^2)z^{\frac{1}{2}}dz$

$= \int_0^1 \left(z^{\frac{1}{2}} - z^{\frac{5}{2}}\right)dz$

$= \left[\frac{2}{3}z^{\frac{3}{2}} - \frac{2}{7}z^{\frac{2}{2}}\right]_0^1$

$= \left(\frac{2}{3} - \frac{2}{7}\right) = \frac{8}{21}$

> ধরি, $\sin x = z$
>
> $\therefore \cos xdx = dz$
>
> $x = 0$ হলে, $z = 0$
>
> $x = \frac{\pi}{2}$, হলে, $z = 1$

(viii) Evaluate: $\int_0^{\pi/2} \sqrt{\cos x}\sin^3 xdx$

[SB. 13; JB. 10; CtgB, 01, 09; BB. 13; RB. 08; CB. 13]

Solution: $\int_0^{\pi/2} \sqrt{\cos x}\sin^3 xdx$

$= \int_0^{\pi/2} \sqrt{\cos x}\sin^2 x \cdot \sin xdx$

$= \int_0^{\pi/2} \sqrt{\cos x}(1-\cos^2 x) \cdot \sin x\, dx$

$=\int_1^0 \sqrt{z}(1-z^2)(-dz)$

$= -\int_1^0 \left(z^{\frac{1}{2}} - z^{\frac{5}{2}}\right)dz$

$= -\left[\frac{2}{3}z^{\frac{3}{2}} - \frac{2}{7}z^{\frac{7}{2}}\right]_1^0$

$= -\left(\frac{2}{7} - \frac{2}{3}\right) = \frac{2}{3} - \frac{2}{7}$

$= \frac{8}{21}$

> ধরি, $\cos x = z$
>
> $\Rightarrow -\sin xdx = dz$
>
> $x = 0$ হলে, $z = 1$
>
> $x = \frac{\pi}{2}$ হলে, $z = 0$

9i)Evaluate: $\int_1^3 \frac{1}{x}\cos(\ln x)dx$ }

[DB. 08; CB, 14, 13, 08; JB 12]

Solution:

$\int_1^3 \frac{1}{x}\cos(\ln x)dx$

$= \int_0^{\ln 3} \cos z\, dz$

$[\sin z]_0^{\ln 3}$

$= \sin(\ln 3) - 0 = \sin(\ln 3)$

> ধরি, ln x=z
>
> $\therefore \frac{1}{x}dx = dz$
>
> $x - 1$ হলে, $z = 0$
>
> $x = 3$ হলে, $z = \ln 3$

9(ii) Evaluate: $\int_1^2 \frac{dx}{x(1+\ln x)^2}$

[DjB. 14; CB.12, 13; JB, 10. 12:DB. 14, RB. 13]

Solution: $\int_1^2 \frac{dx}{x(1+\ln x)^2}$

$= \int_1^3 \frac{1}{z^2}dz$

$= \left[-\frac{1}{z}\right]_1^3$

$= \left(-\frac{1}{3}\right) - (-1)$

$= 1 - \frac{1}{3} = \frac{2}{3}$

> ধরি, $1 + \ln x = 2$
>
> $\therefore \frac{1}{x}dx = dz$
>
> $x = 1$ হলে, $x = 1$
>
> $x = e^2$ হলে, $z = 3$

9(iii) Evaluate:

$\int_1^2 x^2 e^{x^3}dx$

[BB. 02,10; RB, 04,06; DB, 01}

Solation: $\int_1^2 x^2 e^{x^3}dx$

$= \int_1^8 \frac{1}{3}e^z dz$

$= \frac{1}{3}[e^z]_1^8$

$= \frac{1}{3}(e^8 - e^1) = \frac{1}{3}(e^8 - e)$

> ধরি,, $x^3 = z$
>
> $3x^2dx = dz$
>
> $\therefore x^2dx = \frac{1}{3}dz$
>
> $x = 1$ হলে, $z = 1$
>
> $x = 2$ হলে, $z = 8$

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9(iv) Evaluate: $\int_0^1 xe^{x^2}dx$ [DB, 09,13; CB, 12, 13; CtgB. 06,12; DjB. 13, 12; SB, 03, 07, 10]

Solution: $\int_0^1 xe^{x^2}dx$

$= \int_0^1 \frac{1}{2}e^z dz$

$= \frac{1}{2}[e^z]_0^1$

$= \frac{1}{2}(e^1 - e^0)$

$= \frac{1}{2}(e-1)$

ধরি, $x^2 = z$
$\Rightarrow 2xdx = dz$
$\therefore xdx = \frac{1}{2}dz$

$x = 0$ হলে, $z = 0$
$x = 1$ হলে, $z = 1$

10(i) Evaluate: $\int_0^1 \frac{dx}{e^x+e^{-x}}$ [DB. 14; BB.13; RB. 03,12; CB. 08; SB. 07; BB. 12]

Solution: $\int_0^1 \frac{dx}{e^x+e^{-x}}$

$= \int_0^1 \frac{e^x dx}{(e^x)^2+1}$ [e^x দ্বারা লব ও হরকে গুন করি]

$\therefore \int_0^1 \frac{dz}{z^2+1}$

$= [\tan^{-1} z]_1^e$

$= \tan^{-1} e - \tan^{-1} 1$

$= \tan^{-1} e - \frac{\pi}{4}$

ধরি, $e^x = z \therefore e^x dx = dz$
$x = 0$ হলে, $z = e^0 = 1$
$x = 1$ হলে, $z = e^1 = e$

10(ii)Evaluate: $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx$ [DjB. 09, BB. 08; SB. 07; JB. 04]

Solution: $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx$

$= \int_0^{\pi/2} zdz$

$= \left[\frac{1}{2}z^2\right]_0^{\pi/2}$

$= \frac{1}{2}\left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{8}$

ধরি, $\sin^{-1} x = z$
$\therefore \frac{dx}{\sqrt{1-x^2}} = dz$
$x = 0$ হলে, $z = 0$
$x = 1$ হলে, $z = \frac{\pi}{2}$

10(iii) Evaluate: $\int_0^1 \frac{\tan^{-1} x}{1+x^2}dx$ [JB.13: BUET.09-10]

Solution: $\int_0^1 \frac{\tan^{-1} x}{1+x^2}dx$

$= \int_0^{\pi/4} zdz$

$= \left[\frac{z^2}{2}\right]_0^{\frac{\pi}{4}}$

$= \frac{\left(\frac{\pi}{4}\right)^2}{2} - 0 = \frac{\pi^2}{32}$

ধরি, $\tan^{-1} x = z$
$\therefore \frac{dx}{1+x^2} = dz.$
$x = 1$ হলে, $z = \frac{\pi}{4}$
x = 0 হলে, Z = 0

10(iv) Evaluate: $\int_0^1 \frac{(\tan^{-1} x)^2}{1+x^2}dx$ [BB, 06,12; DB, 05, 11; CB. 11, 13;JB, 10; SB, 06,10]

Solution: $\int_0^1 \frac{(\tan^{-1} x)^2}{1+x^2}dx.$

$= \int_0^{\pi/4} z^2 dz$

$= \left[\frac{z^3}{3}\right]_0^{\frac{\pi}{4}}$

$= \frac{\left(\frac{\pi}{4}\right)^3}{3} - 0$

ধরি, $\tan^{-1} x = z$
$\therefore \frac{dx}{1+x^2} = dz.$
$x = 1$ হলে, $z = \frac{\pi}{4}$
x = 0 হলে, Z = 0

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11 (i)Evaluate: $\int_0^4 y\sqrt{4-y}\,dy$
[SB. 14; DB. 10,12; CtgB, 10, 14; DjB. 10; RB. 07; BB. 05; RB. 13]

Solution: $\int_0^4 y\sqrt{4-y}dy$

$= \int_2^0 (4-z^2)\cdot z(-2zdz)$

$= -2\int_2^0 (4-z^2)z^2dz$

$=-2\int_2^0 (4z^2-z^4)\,dz$

$= -2\left[\frac{4z^3}{3}-\frac{z^5}{5}\right]_2^0$

= - 2

ধরি, $4-y=z^2$
$\Rightarrow y = 4-z^2$
$\therefore dy = -2z\,dz$
$y=0$ হলে, $z=2$
$y=4$ হলে, $z=0$

11(ii)Evaluate: $\int_0^4 \sqrt{16-x^2}dx$
[RB. 14; BB, 14; CB, 03, 11; SB. 09, 11, 13; JB, 05]

Solution: $\int_0^4 \sqrt{16-x^2}dx$

$= \int_0^{\pi/2} \sqrt{16-16\sin^2\theta}\,4\cos\theta d\theta$

$= \int_0^{\pi/2} \sqrt{16(1-\sin^2\theta)}4\cos\theta d\theta$

$= \int_0^{\pi/2} 4\sqrt{\cos^2\theta}\cdot 4\cos\theta d\theta$

$= \frac{16}{2}\int_0^{\pi 2} 2\cos^2\theta d\theta$

$= 8\int_0^{\pi/} (1+\cos 2\theta)d\theta$

$= 8\left[\theta+\frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$

$= 8\left\{\left(\frac{\pi}{2}+\frac{1}{2}\sin\pi\right)-0\right\}$

$= 4\pi$

ধরি, $x = 4\sin\theta$
$\therefore dx = 4\cos\theta d\theta$
$x=0$ হলে, $\theta=0$
$x=4$ হলে, $\theta=\frac{\pi}{2}$

11(iii)Evaluate: $\int_{-1}^1 x^2\sqrt{4-x^2}dx$
[JB. 05, 09; DB. 08; BB. 08; CtgB, 03]

Solution:
$\int_{-1}^1 x^2\sqrt{4-x^2}dx$

ধরি, $x = 2\sin\theta$
$\therefore dx = 2\cos\theta d\theta$
$x=-1$ হলে, $\theta = -\frac{\pi}{6}$
$x=1$ হলে, $\theta=\frac{\pi}{6}$

$= \int_{-\pi/6}^{\pi/6} 4\sin^2\theta\sqrt{4-4\sin^2\theta}\cdot 2\cos\theta d\theta$

$= \int_{-\pi}^{\pi/6} 16\sin^2\theta\cos^2\theta d\theta$

$= 4\int_{-\pi/6}^{\pi/6} (2\sin\theta\cos\theta)^2 d\theta$

$=4\int_{-\pi/6}^{\pi/6} (\sin 2\theta)^2$

$=2\int_{-\pi/6}^{\pi/6} 2\sin^2 2\theta$

$=2\int_{-\pi/6}^{\pi/6} (1-\cos 4\theta)$

$=2\left[\theta+\frac{\sin 4\theta}{4}\right]_{-\pi/6}^{\frac{\pi}{6}}$

$=2[\frac{\pi}{6}+\frac{\sin\frac{4\pi}{6}}{4} - \{-\frac{\pi}{6} - \frac{\sin\frac{4\pi}{6}}{4}\}]$

$=2\,[\frac{\pi}{6}+2\sin\frac{2\pi}{3}]$

11(iv)Evaluate: $\int_1^2 \frac{dx}{x^2\sqrt{4-x^2}}$ [RUET, 04-05]

Solution:
$\int_1^2 \frac{dx}{x^2\sqrt{4-x^2}}$

ধরি, $x = 2\sin\theta$
$\therefore dx = 2\cos\theta d\theta$
$x=1$ হলে, $\theta=\frac{\pi}{6}$
$x=2$ হলে, $\theta=\frac{\pi}{2}$

$= \int_{\pi/6}^{\pi/2} \frac{2\cos\theta d\theta}{(2\sin\theta)^2\cdot\sqrt{4-4\sin^2\theta}}$

$= \int_{\pi/6}^{\pi/2} \frac{2\cos\theta d\theta}{4\sin^2\theta.2\cos\theta}$

$=\frac{1}{4}\int_{\pi/6}^{\pi/2}\text{cosec}^2\theta$

$=\frac{1}{4}[-\cot\ \theta]_{\pi/6}^{\pi/2}$

$=-\frac{1}{4}\left[\cot\frac{\pi}{2}-\cot\frac{\pi}{6}\right]$

$=-\frac{1}{4}[0-\sqrt{3}]=\frac{\sqrt{3}}{4}$

11(v)Evaluate: $\int_1^{\sqrt{e}}x\ln\ x dx$

Solution: $\int x\ln\ x dx$

$=\ln\ x\int xdx-\int\left\{\frac{d}{dx}(\ln\ x)\int xdx\right\}dx$

$=\frac{x^2}{2}\ln\ x-\int\frac{1}{x}\cdot\frac{x^2}{2}dx$

$=\frac{x^2}{2}\ln\ x-\int\frac{x}{2}dx$

$=\frac{x^2}{2}\ln\ x-\frac{1}{2}\cdot\frac{x^2}{2}=\frac{1}{2}\left(x^2\ln\ x-\frac{x^2}{2}\right)$

$\therefore\int_1^{\sqrt{e}}x\ln\ x dx=\frac{1}{2}\left[x^2\ln\ x-\frac{x^2}{2}\right]_1^{\sqrt{e}}$

$=\frac{1}{2}\left\{\left(e\cdot\ln\ \sqrt{e}-\frac{e}{2}\right)-\left(0-\frac{1}{2}\right)\right\}$

$=\frac{1}{2}\left\{\left(\frac{1}{2}e-\frac{e}{2}\right)+\frac{1}{2}\right\}=\frac{1}{4}$

11(vi) Evalaate: $\int_2^4\ln 2x\ dx$ **[BB, 14,09; JB. 34]**

Solufion: $\int\ln 2x\ dx$

$=\ln 2x\int dx-\int\left\{\frac{d}{dx}(\ln\ 2x)\int dx\right\}dx$

$=x\ln 2x-\int\left(\frac{1}{2x}\times 2x\right)dx=x\ln 2x-x$

$\therefore\int_2^4\ln 2x\,dx=[x\ln 2x-x]_2^4$

$=(4\ln\ 8-4)-(2\ln\ 4-2)$

$=4\ln\ 2^3-4-2\ln\ 2^2+2$

$=12\ln\ 2-4\ln\ 2-2$

$=8\ln\ 2-2$

11(vii) Evaluate: $\int_0^1\ln\ (x^2+$ $1)dx$ **[CtgB, 14; DB. 07]**

Solution: $\int\ln\ (x^2+1)dx$

$=\ln\ (x^2+1)\int dx-\int\left(\frac{d}{dx}\ln\ (x^2+1)\int dx\right\}dx$

$=x\ln(x^2+1)-\int\frac{1}{x^2+1}.2x\cdot xdx$

$=x\ln\ (x^2+1)-\int\frac{2x^2}{x^2+1}dx$

$=x\ln\ (x^2+1)-2\int\frac{(x^2+1)-1}{x^2+1}dx$

$=x\ln\ (x^2+1)-2\int\left(1-\frac{1}{1+x^2}\right)dx$

$=x\ln\ (x^2+1)-2x+2\tan^{-1}\ x$

$\therefore\int_0^1\ln\ (x^2+1)dx$

$=[x\ln\ (x^2+1)-2x+2\tan^{-1}\ x]_0^1$

$=\ln\ 2-2+2\tan^{-1}\ 1-0=\ln\ |2|-2+2\frac{\pi}{4}$

$=\ln 2\ -2+\frac{\pi}{2}$

11(viii) Evaluate: $\int_1^{\sqrt{3}}\mathbf{x\tan^{-1}\ xdx}$ **|CB. 14 ; RB. 08, 12, 13; CtgB. 08; 12; DjB.12]**

Solution: $\int x\tan^{-1}\ xdx$

$=\tan^{-1}\ x\int xdx-\int\left\{\frac{d}{dx}(\tan^{-1}\ x)\int xdx\right\}dx$

$=\frac{1}{2}x^2\tan^{-1}\ x-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$

$=\frac{1}{2}x^2\tan^{-1}\ x-\frac{1}{2}\int\frac{(1+x^2-1)}{1+x^2}dx$

$=\frac{1}{2}x^2\tan^{-1}\ x-\frac{1}{2}\int dx+\frac{1}{2}\int\frac{dx}{1+x^2}$

$=\frac{1}{2}x^2\tan^{-1}\ x-\frac{1}{2}x+\frac{1}{2}\tan^{-1}\ x$

$\therefore\int_1^{\sqrt{3}}x\tan^{-1}\ xdx$

$=\left[\frac{1}{2}x^2\tan^{-1}\ x-\frac{1}{2}x+\frac{1}{2}\tan^{-1}\ x\right]_1^{\sqrt{3}}$

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$= \left(\frac{1}{2}\cdot 3\tan^{-1}\sqrt{3} - \frac{1}{2}\cdot\sqrt{3} + \frac{1}{2}\tan^{-1}\sqrt{3}\right)$

$-\left(\frac{1}{2}\tan^{-1} 1 - \frac{1}{2} + \frac{1}{2}\tan^{-1}\right)$

$=\left(\frac{3}{2}\frac{\pi}{3} - \frac{\sqrt{3}}{2} + \frac{1}{2}\frac{\pi}{3}\right) - \left(\frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{2} + \frac{1}{2}\frac{\pi}{4}\right)$

$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{5\pi}{12} - \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{5\pi}{12} - \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1}{12}(5\pi - 6\sqrt{3} + 6)$

11 (ix) Evaluate: $\int_1^{\sqrt{3}} x\cot^{-1} x\, dx$

Solution: $\int x\cot^{-1} x\, dx$

$= \cot^{-1} x\int xdx - \int\left\{\frac{d}{dx}(\cot^{-1} x)\int xdx\right\}dx$

$= \frac{1}{2}x^2\cot^{-1} x + \frac{1}{2}\int \frac{x^2}{1+x^2}dx$

$= \frac{1}{2}x^2\cot^{-1} x + \frac{1}{2}\int \frac{(1+x^2-1)}{1+x^2}dx$

$= \frac{1}{2}x^2\cot^{-1} x + \frac{1}{2}\int dx - \frac{1}{2}\int \frac{dx}{1+x^2}$

$= \frac{1}{2}x^2\cot^{-1} x + \frac{1}{2}x - \frac{1}{2}\tan^{-1} x$

$\therefore \int_1^{\sqrt{3}} x\cot^{-1} xdx$

$= \left[\frac{1}{2}x^2\cot^{-1} x + \frac{1}{2}x - \frac{1}{2}\tan^{-1} x\right]_1^{\sqrt{3}}$

$= \left(\frac{1}{2}\cdot 3\cot^{-1}\sqrt{3} + \frac{1}{2}\cdot\sqrt{3} - \frac{1}{2}\tan^{-1}\sqrt{3}\right)$

$-\left(\frac{1}{2}\cot^{-1} 1 + \frac{1}{2} - \frac{1}{2}\tan^{-1} 1\right)$

$= \left(\frac{3}{2}\frac{\pi}{6} + \frac{\sqrt{3}}{2} - \frac{1}{2}\frac{\pi}{3}\right) - \left(\frac{1}{2}\frac{\pi}{4} + \frac{1}{2} - \frac{1}{2}\frac{\pi}{4}\right)$

$= \frac{\pi}{4} + \frac{\sqrt{3}}{2} - \frac{\pi}{6} - \frac{1}{2} = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - \frac{1}{2}$

12(i) Evaluate: $\int_0^3 \frac{xe^x}{3(x+1)^2}dx$ [JB. 17; RB,14]

Solution: $\int_0^3 \frac{xe^x}{3(x+1)^2}dx$

$= \frac{1}{3}\int_0^3 \left\{e^x\frac{x+1-1}{(x+1)^2}\right\}dx$

$= \frac{1}{3}\int_0^3 e^x\left\{\frac{1}{x+1} + \frac{-1}{(x+1)^2}\right\}dx$

$= \frac{1}{3}\left[\frac{e^x}{(x+1)}\right]_0^3$

$= \frac{1}{3}\left\{\frac{e^3}{3+1} - \frac{e^0}{(0+1)^2}\right\}$

$= \frac{1}{3}\left(\frac{e^3}{4} - 1\right)$

12(ii) Evaluate: $\int_2^e \left\{\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right\}dx$ [BUET.03]

Solution: $\int_2^e \left[\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right]dx$

Now $\int \left[\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right]dx$

$= \int \frac{dx}{\ln x} - \int \frac{1}{(\ln x)^2}dx$

$= \frac{1}{\ln x}\int dx - \int\left\{\frac{d}{dx}\left(\frac{1}{\ln x}\int dx\right)\right\}dx$

$-\int \frac{1}{(\ln x)^2}dx$

$= \frac{x}{\ln x} - \int \frac{-1}{(\ln x)^2\cdot x}\cdot xdx - \int \frac{1}{(\ln x)^2}dx$

$= \frac{x}{\ln x} + \int \frac{dx}{(\ln x)^2} - \int \frac{dx}{(\ln x)^2} = \frac{x}{\ln x}$

$\therefore \int_2^e \left[\frac{1}{\ln x} - \frac{1}{(\ln x)^2}\right]dx = \left[\frac{x}{\ln x}\right]_2^e$

$= \frac{e}{\ln e} - \frac{2}{\ln 2} = e - \frac{2}{\ln 2}$

13(i)Evaluate: $\int_{1/2}^1 \frac{dx}{x\sqrt{4x^2-1}}$ [BUET. 04-05]

Solution: ধরি,

$4x^2 - 1 = z^2$

$\Rightarrow 4x^2 = z^2 + 1$

$\therefore 8xdx = 2zdz \Rightarrow 4xdx = zdz$

$x=\frac{1}{2}$ হলে, $z=0$

$x=1$ হলে, $z=\sqrt{3}$

$$\int_{1/2}^{1}\frac{dx}{x\sqrt{4x^2-1}}=\int_{1/2}^{1}\frac{4xdx}{4x^2\sqrt{4x^2-1}}$$

$$=\int_0^{\sqrt{3}}\frac{zdz}{(z^2+1)z}$$

$$=\int_0^{\sqrt{3}}\frac{1}{1+z^2}dz=[\tan^{-1} z]_0^{\sqrt{3}}$$

$$=\tan^{-1}(\sqrt{3})-\tan^{-1} 0=\frac{\pi}{3}-0=\frac{\pi}{3}$$

Exercise-10.7

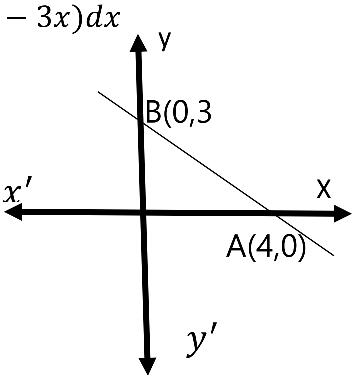
প্রশ্ন 01। $3x+4y=12$ **সরল রেখা এবং স্থানাঙ্কের অক্ষ দ্বয় দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। (M.R.'03)**

Solution: প্রদত্ত সরলরেখার সমীকরণ

$3x+4y=12$ or, $4y=12-3x$ or. $y=\frac{1}{4}(12-3x)$

সরল রেখাটি x অক্ষকে (4,0) বিন্দুতে এবং y অক্ষকে (0,3) বিন্দুতে ছেদ করে,

$$\triangle OAB=\int_0^4 ydx=\frac{1}{4}\int_0^4(12-3x)dx$$



$$=\frac{1}{4}\int_0^4 12\,dx-\frac{1}{4}\int_0^4 3xdx$$

$$=3[x]_a^4-\frac{3}{4}\left[\frac{x^2}{2}\right]_0^4$$

$$=3[4-0]-\frac{3}{4}\left[\frac{4^2}{2}-\frac{0^2}{2}\right]$$

$$=12-\frac{3}{4}[8]=12-6=6$$

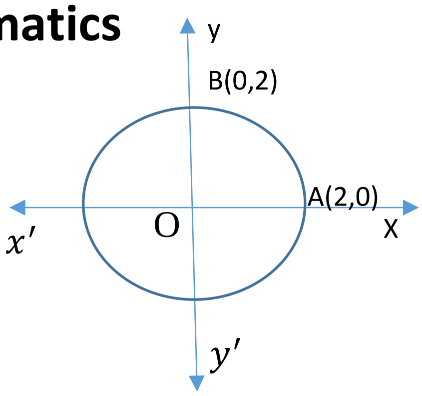
∴ নির্ণেয় ক্ষেত্রফল $=6$ বর্গ একক (Ans.)

প্রশ্ন 02। $x^2+y^2=4$**. বৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। (D.B.'07)**

Solution: $x^2+y^2=4$ বৃত্তের কেন্দ্র (0,0) এবং ব্যাসার্ধ 2 .

প্রদত্ত বৃত্তের সমীকরণ $x^2+y^2=4$

or, $y=\pm\sqrt{4-x^2}$



∴ OAB ক্ষেত্রের ক্ষেত্রফল

$$=\int_0^2 ydx$$

$$=\int_0^2\sqrt{2^2-x^2}dx$$

$$=\left[\frac{x\sqrt{2^2-x^2}}{2}+\frac{2^2}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$=\frac{2\sqrt{2^2-2^2}}{2}+\frac{2^2}{2}\sin^{-1}\frac{2}{2}-\frac{0\sqrt{2^2-0^2}}{2}-\frac{0^2}{2}\sin^{-1}\frac{0}{4}$$

$$=2\sin^{-1}1$$

$$=2\frac{\pi}{2}$$

$$=\pi$$

∴সমগ্র বৃত্তের ক্ষেত্রফল $=4\times$ OAB ক্ষেত্রের ক্ষেত্রফল

$=4\pi$ বর্গ একক (Ans.)

প্রশ্ন 03। $x^2+y^2=16$ **বৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।**

[J.B.'14; S.B.'14;Di.B.'12: D.B.'12: C.B.'11,07,00: B.B.'11, '08, '06]

Solution প্রদত্ত বৃত্তের সমীকরণ

$x^2+y^2=16$

or, $x^2+y^2=4^2$ বৃত্তের কেন্দ্র (0,0) এবং ব্যাসার্ধ 4..

আবার, $x^2+y^2=16$

or, $y^2=16-x^2$

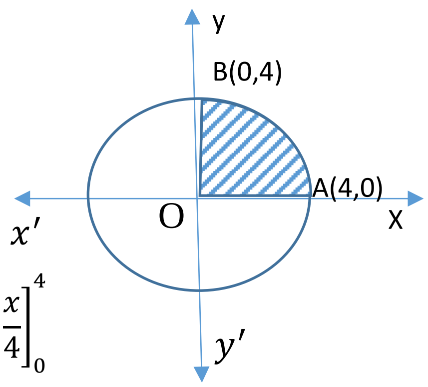
or, $y=\sqrt{16-x^2}$

$\therefore$ OAB ক্ষেত্রের ক্ষেত্রফল,

$= \int_0^4 y dx$

$= \int_0^4 \sqrt{16 - x^2} dx$

$= \int_0^4 \sqrt{4^2 - x^2} dx$



$= \left[\frac{x\sqrt{4^2 - x^2}}{2} + \frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$= \frac{4\sqrt{4^2 - 4^2}}{2} + \frac{4^2}{2}\sin^{-1}\frac{4}{4} - \frac{0\sqrt{4^2 - 0^2}}{2} - \frac{0^2}{2}\sin^{-1}\frac{0}{4}$

$= \frac{16}{2}\sin^{-1} 1 = 8 \cdot \frac{\pi}{2} = 4\pi$

সমগ্র বৃত্তের ক্ষেত্রফল $= 4 \times$ OAB ক্ষেত্রের ক্ষেত্রফল

$= 4 \times 4\pi$ বর্গ একক

$= 16\pi$ বর্গ একক (Ans.)

প্রশ্ন 04। $x^2 + y^2 = 36$বৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। .[B.B · 17]

Solution: প্রদত্ত বৃত্তের সমীকরণ $x^2 + y^2 = 36$

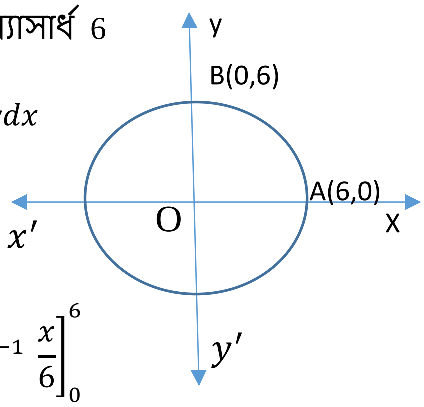
or. $x^2 + y = 6^2$ ..., (i)

বৃত্তের কেন্দ্র (0,0) এবং ব্যাসার্ধ 6

বৃত্তের ক্ষেত্রফল $= 4 \times \int_0^6 y dx$

$= 4\int_0^6 \sqrt{36 - x^2} dx$

$= 4\int_0^6 \sqrt{6^2 - x^2} dx$



$= 4\left[\frac{x\sqrt{6^2 - x^2}}{2} + \frac{6^2}{2}\sin^{-1}\frac{x}{6}\right]_0^6$

$= 4[\frac{6\sqrt{6^2-6^2}}{2} + \frac{6^2}{2}\sin^{-1}\frac{6}{6} - \frac{0\sqrt{6^2-0^2}}{2} - \frac{0^2}{2}\sin^{-1}\frac{0}{6}]$

$= 4.\frac{36}{2}\sin^{-1} 1 = 72 \cdot \frac{\pi}{2} = 36\,\pi$ বর্গ একক (Ans.)

প্রশ্ন 05।$2x^2 + 2y^2 = 64$ দ্বারা প্রথম চতুর্ভাগের আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। [D.B.'17]

Solution: প্রদত্ত বৃত্তের সমীকরণ

$2x^2 + 2y^2 = 64$

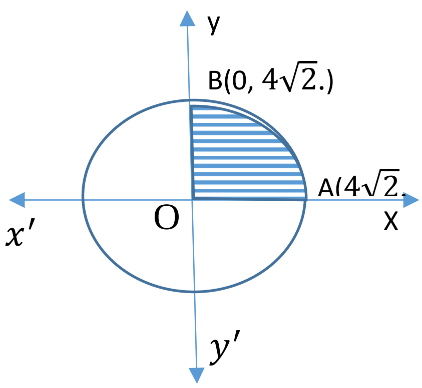
$x^2 + y^2 = 32$

$x^2 + y^2 = (4\sqrt{2})^2$

প্রদত্ত বৃত্তের সমীকরণ বৃত্তের কেন্দ্র (0,0) এবং ব্যাসার্ধ $= 4\sqrt{2}$.

প্রদত্ত বৃত্তের সমীকরণ থেকে পাই, $2x^2 + 2y^2 = 64$

$\Rightarrow y = \sqrt{32 - x^2}$



ধরি, $x = 4\sqrt{2}\sin\,\theta$

$\therefore$ dx $= 4\sqrt{2}\cos\,\theta d\theta$

$x = 0$ হলে, $\sin\,\theta = 0$

$\therefore \theta = 0$

$x = 4\sqrt{2}$হলে

$\sin\,\theta = 1$

$\therefore \theta = \frac{\pi}{2}$

$= \int_0^{4\sqrt{2}} y dx$

$= \int_0^{4\sqrt{2}} \sqrt{32 - x^2} dx$

$= \int_0^{\frac{5}{2}} \sqrt{32-32\sin^2\,\theta} \cdot 4\sqrt{2}\cos\,\theta d\theta$

$= 32\int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2\,\theta} \cdot \cos\,\theta d\theta$

$= 32\int_0^{\frac{\pi}{2}} \cos^2\,\theta d\theta$

$= 16\int_0^{\frac{\pi}{2}} 2\cos^2\,\theta d\theta$

$= 16\int_0^{\frac{\pi}{2}} (1 + \cos\,2\theta) d\theta$

$= 16\left[\theta + \frac{\sin\,2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= 16\left[\left(\frac{\pi}{2} + \frac{\sin\,\pi}{2}\right) - 0\right]$

$= 8\pi$বর্গ একক (Ans.)

প্রশ্ন 06।$x^2 + y^2 = 36$ বৃত্তের x অক্ষের উপরের অংশের ক্ষেত্রফল সমাকলন পদ্ধতিতে নির্ণয় কর। [J.B.19]

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Solution: প্রদত্ত বৃত্তের সমীকরণ

. $x^2 + y^2 = 36 \ldots\ldots\ldots\ldots\ldots(i)$

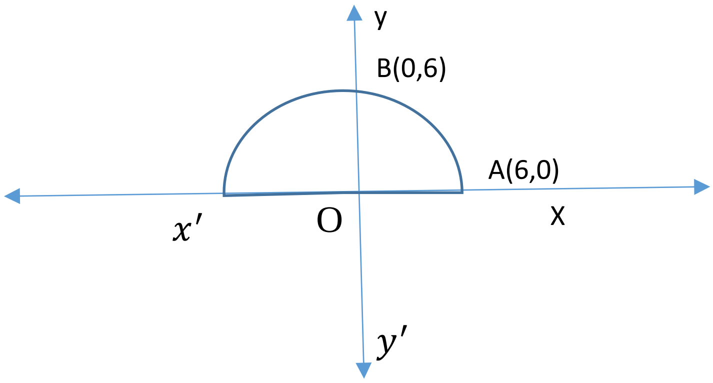
or, $x^2 + y^3 = 6^2$

বৃত্তের কেন্দ্র (0,0) এবং ব্যাসার্ধ 6

(i),থেকে পাই , $x^2 + y^2 = 36$

$$\text{Or. } y^2 = 36 - x^2$$

$$\therefore y = \sqrt{36 - x^2}$$



∴ OAB ক্ষেত্রের ক্ষেত্রফল

$$= \int_0^6 ydx$$

$$= \int_0^6 \sqrt{36 - x^2}dx$$

$$= \int_0^6 \sqrt{6^2 - x^2}dx$$

$$= \left[\frac{x\sqrt{6^2 - x^2}}{2} + \frac{6^2}{2}\sin^{-1}\frac{x}{6}\right]_0^{10}$$

$$= \frac{6\sqrt{6^2 - 6^2}}{2} + \frac{36}{2}\sin^{-1}\frac{6}{6} - 0 \cdot \frac{36}{2}\sin^{-1}0$$

$$= 0 + 18\sin^{-1}1 - 0 = 18\sin^{-1}\sin\frac{\pi}{2}$$

$= 18.\frac{\pi}{2} = 9\pi$ বর্গ একক

অর্ধবৃত্তের ক্ষেত্রফল

$= 2\int_0^6 ydx$

$= 2 \times 9\pi$

$= 18\pi$ বর্গ একক (Ans.)

প্রশ্ন 07। $\frac{x^2}{16} + \frac{y^2}{4} = 1$ উপবৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

[J.B."11: C.B.09 S.B:'13: B.B.'12,'09,07; D.B:11: R.B.'14.' 12. 06, D.B.12.Ctg.B, 12,06]

Solution: **প্রদত্ত উপবৃত্তের সমীকরণ**

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

$\therefore \frac{x^2}{4^2} + \frac{x^2}{2^2} = 1 \ldots \ldots.(i)$

উপবৃত্তের কেন্দ্র (0,0) এবং

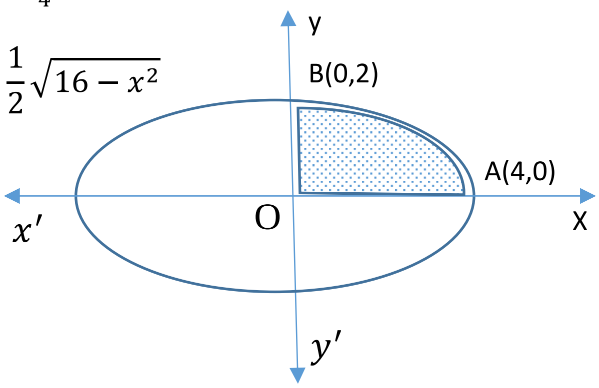
উপবৃত্তের বৃহৎ অক্ষের দৈর্ঘ্য = 8 একক

উপবৃত্তের ক্ষুদ্র অক্ষের দৈর্ঘ্য = 4 একক

(i)নং সমীকরণ থেকে পাই , $\frac{y^2}{4} = 1 - \frac{x^2}{16}$

or, $y^2 = \frac{1}{4}(16 - x^2)$

$$\therefore y = \frac{1}{2}\sqrt{16 - x^2}$$



এখানে, (i) নং উপবৃত্তটি চারটি চতুর্ভাগে সমান অংশে বিভক্ত ও প্রথম চতুর্ভাগে X এর সীমা 0 থেকে 4 পর্যন্ত ।

∴ OAB এর ক্ষেত্রের ক্ষেত্রফল

$= \int_0^4 ydx$

$= \frac{1}{2}\int_0^4 \sqrt{16 - x^2}dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \sqrt{16 - 16\sin^2\theta}.4\cos\theta d\theta$

$= \frac{1}{2}\int_0^{\frac{\pi}{2}}.4\cos\theta\, 4\cos\theta d\theta$

ধরি, $x = 4\sin\theta$

$\therefore dx = 4\cos\theta d\theta$

$x = 0$ হলে, $\theta = 0$

$x = 4$ হলে, $\theta = \frac{\pi}{2}$

$= 4\int_0^{\frac{\pi}{2}} 2\cos^2 \theta d\theta$

$= 4\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta)d\theta$

$= 4.\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= 4\left[\left(\frac{\pi}{2} + \frac{\sin \pi}{2}\right) - 0\right]$

$= 2\pi$

$\therefore$ উপবৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= 4 \times$ (OAB এর ক্ষেত্রফল)

$= 4 \times 2\pi$

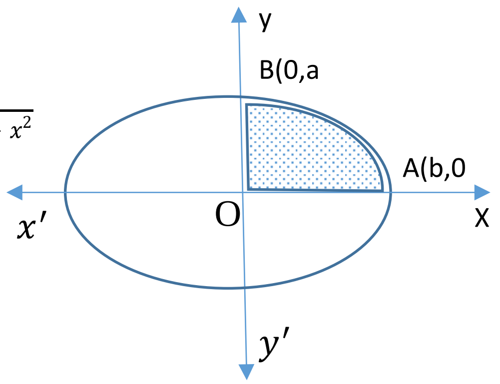
$= 8\pi$ বর্গ একক (Ans.)

প্রশ্ন 08। b > a হলে $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ বক্ররেখা দ্বারা আবদ্ধ ক্ষেত্রের অর্ধাংশের ক্ষেত্রফল বের কর।
[Di.B17]

Solution: প্রদত্ত বক্ররেখার সমীকরণ

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\therefore y = \frac{a}{b}\sqrt{b^2 - x^2}$



প্রথম চতুর্ভাগে X এর সীমা 0 থেকে b পর্যন্ত।

$\therefore$ নির্ণেয় ক্ষেত্রের ক্ষেত্রফল

$= 2\int_0^b ydx$

$= 2\frac{a}{b}\int_0^b \sqrt{b^2 - x^2}dx$

$= \frac{2a}{b}\int_0^{\frac{\pi}{2}} \sqrt{b^2 - b^2\sin^2 0} \cdot b\cos \theta d\theta$

$= \frac{2a}{b}\int_0^{\frac{\pi}{2}} b\cos \theta \cdot b\cos \theta d\theta$

ধরি, $x = b \sin\theta$

$\therefore dx = b\cos\theta d\theta$

$x = 0$ হলে, $\theta = 0$

$x = b$ হলে, $\theta = \frac{\pi}{2}$

$= \frac{ab^2}{b}\int_0^{\frac{\pi}{2}} 2\cos^2 \theta d\theta$

$= ab\int^{\frac{\pi}{2}} (1 + \cos 2\theta)d\theta$

$= \text{ab}\left[0 + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= ab\left[\left(\frac{\pi}{2} + 0\right) - 0\right]$

$= \frac{\pi ab}{2}$

$\therefore$ নির্ণেয় ক্ষেত্রফল $= \frac{\pi ab}{2}$ বর্গ একক (Ans.)

প্রশ্ন 09। $4x^2 + 9y^2 = 36$ উপবৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

Solution: প্রদত্ত উপবৃত্তের সমীকরণ ,

$4x^2 + 9y^2 = 36$

or, $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$\therefore \frac{x^2}{3^2} + \frac{y^2}{2^2} = 1 \dots\dots \dots$ (i)

উপবৃত্তের কেন্দ্র (0,0) এবং ,

উপবৃত্তের বৃহৎ অক্ষের দৈর্ঘ্য $= 2.3 = 6$একক

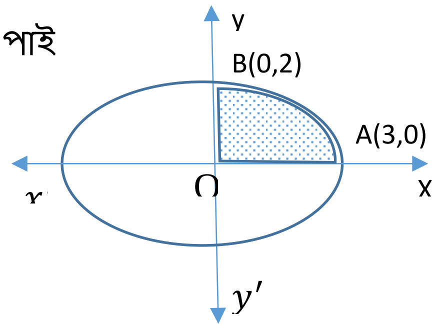
উপবৃত্তের ক্ষুদ্র অক্ষের দৈর্ঘ্য $2.2 = 4$ একক

(i)নং সমীকরণ থেকে পাই

or. $\frac{y^2}{4} = 1 - \frac{x^2}{9}$

or. $y^2 = \frac{4}{9}(9 - x^2)$

$y = \frac{2}{3}\sqrt{9 - x^2}$



এখানে, (i) নং উপবৃত্তটি চারটি চতুর্ভাগে সমান অংশে বিভক্ত ও প্রথম চতুর্ভাগে X এর সীমা 0 থেকে 3 পর্যন্ত।

$\therefore$ OAB এর ক্ষেত্রের ক্ষেত্রফল

$= \int_0^3 ydx$

$= \frac{2}{3}\int_0^3 \sqrt{9 - x^2}dx$

$= 3\int_0^{\frac{\pi}{2}} 2\cos^2 \theta d\theta$

$= 3\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta)d\theta$

$= 3.\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

ধরি, $x = 3 \sin\theta$

$\therefore dx = 3\cos\theta d\theta$

$x = 0$ হলে, $\theta = 0$

$x = 3$ হলে, $\theta = \frac{\pi}{2}$

$= 3\cdot\left[\left(\frac{\pi}{2}+\frac{\sin\ \pi}{2}\right)-0-\frac{1}{2}\sin\ 0\right]$

$= \frac{3\pi}{2}$ বর্গ একক

$\therefore$ উপবৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= 4\times$ (OAB এর ক্ষেত্রফল)

$= 4\cdot\frac{3}{2}\pi$

$= 6\pi$ বর্গ একক (Ans.)

বিকল্প পদ্ধতিঃ

$\therefore$ OAB এর ক্ষেত্রের ক্ষেত্রফল

$= \int_0^3 ydx$

$= \frac{2}{3}\int_0^3\sqrt{9-x^2}dx$

$= \frac{2}{3}\cdot\left[\frac{x\cdot\sqrt{9-x^2}}{2}+\frac{3^2}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$

$= \frac{2}{3}\left[\left\{0+\frac{9}{2}\sin^{-1}(1)\right\}-0\right]=\frac{2}{3}\cdot\frac{9}{2}\cdot\frac{\pi}{2}=\frac{3\pi}{2}$

$\therefore$ উপবৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= 4\times$ (OAB এর ক্ষেত্রফল)

$= 4\cdot\frac{3}{2}\pi$

$= 6\pi$ বর্গ একক (Ans.)

প্রশ্ন 10। উপবৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। $9x^2+25y^2=225$ [D. B. ' 19]

Solution: $9x^2+25y^2=225$

or, $9x^2+25y^2=225$

or, $\frac{x^2}{25}+\frac{y^2}{9}=1$

$\therefore \frac{x^2}{5^2}+\frac{y^2}{3}=1$ ……… (i)

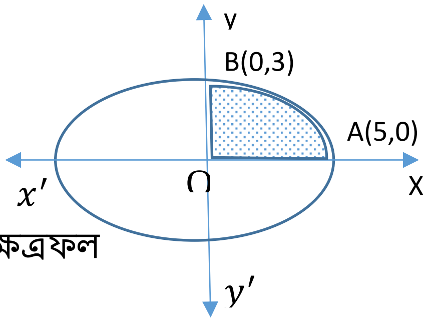
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ কে নং এর সাথে তুলনা করে

$a=5, b=3 \therefore a>b$

উপবৃত্তের কেন্দ্র (0,0) এবং

উপবৃত্তের বৃহৎ অক্ষের দৈর্ঘ্য $2.5=10$ একক

উপবৃত্তের ক্ষুদ্র অক্ষের দৈর্ঘ্য $2.3=6$ একক



or, $\frac{y^2}{9}=1-\frac{x^2}{25}$

or, $y=\frac{3}{5}\sqrt{25-x^2}$

$\therefore$ OAB এর ক্ষেত্রের ক্ষেত্রফল

$= \int_0^5 ydx$

$= \frac{3}{5}\int_0^5\sqrt{25-x^2}dx$ ………… (ii)

ধরি, $x=5\sin\theta$

$\therefore dx=5\cos\theta d\theta$

$x=0$ হলে, $\theta=0$

$x=5$ হলে, $\theta=\frac{\pi}{2}$

$= \frac{3}{5}\int_0^{\frac{\pi}{2}}\sqrt{25-25\sin^2\theta}\cdot 5\cos\theta d\theta$

$= \frac{15}{2}\int_0^{\frac{\pi}{2}} 2\cos^2\theta d\theta$

$= \frac{15}{2}\int_0^{\frac{\pi}{2}}(1+\cos 2\theta)d\theta$

$= \frac{15}{2}\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}=\frac{15}{2}\left[\frac{\pi}{2}+\frac{\sin\pi}{2}-0-\frac{1}{2}\sin 0\right]$

$= \frac{15}{2}.\frac{\pi}{2}=\frac{15\pi}{4}$

উপবৃত্তের ক্ষেত্রফল $= 4$ (OAB এর ক্ষেত্রফল)

$= 4\cdot\frac{15}{4}\pi$

$= 15\pi$ বর্গ একক (Ans.)

(ii) নং থেকে আমরা পাই

OAB এর ক্ষেত্রের ক্ষেত্রফল $= \frac{3}{5}\int_0^5\sqrt{25-x^2}dx$

$= \frac{3}{5}\cdot\left[\frac{x\cdot\sqrt{25-x^2}}{2}+\frac{5^2}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$

$= \frac{3}{5}\left[\left\{0+\frac{25}{2}\sin^{-1}(1)\right\}-0\right]$

$= \frac{3}{5},\frac{25}{2},\frac{\pi}{2}=\frac{5\pi}{4}$

উপবৃত্তের ক্ষেত্রফল $= 4$ (OAB এর ক্ষেত্রফল)

$= 4 \cdot \frac{15}{4}\pi$

$= 15\pi$ বর্গ একক (Ans.)

প্রশ্ন 11। $16x^2 + 25y^2 - 400 = 0$, দ্বারা x অক্ষের উপরিভাগে আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

Solution $16x^2 + 25y^2 - 100 = 0$

or, $16x^2 + 25y^2 = 400$

or, $\frac{x^2}{25} + \frac{y^2}{16} = 1$

$\therefore \frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$

$\therefore$ উপবৃত্তের কেন্দ্র (0,0) এবং

উপবৃত্তের বৃহৎ অক্ষের দৈর্ঘ্য $2.5 = 10$ একক

উপবৃত্তের ক্ষুদ্র অক্ষের দৈর্ঘ্য $2.4 = 8$ একক

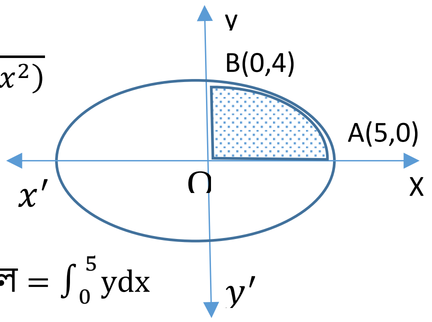
$16x^2 + 25y^2 - 400 = 0$

or, $25y^2 = 400 - 16x^2$

or, $y^2 = \frac{1}{25}(400 - 16x^2)$

$\therefore y = \frac{1}{5}\sqrt{16(25 - x^2)}$

$= \frac{4}{5}\sqrt{25 - x^2}$



$\therefore$ OAB এর ক্ষেত্রফল $= \int_0^5 y\,dx$

$= \frac{4}{5}\int_0^5 \sqrt{25 - x^2}\,dx$

$= \frac{4}{5}\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{5^2}{2}\sin^{-1}\frac{x}{5}\right]_0^5$

$= \frac{4}{5}\left\{\left(0 + \frac{25}{2}\sin^{-1} 1\right) - 0\right\}$

$= \frac{4}{5} \cdot \frac{25}{2} \cdot \frac{\pi}{2} = 5\pi$

$\therefore$ x অক্ষের উপরিভাগে আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

$= 2 \times$ OAB এর ক্ষেত্রফল

$= 2 \times 5\pi = 10\pi$ বর্গ একক (Ans.)

প্রশ্ন 12। $x^2 + y^2 = 25$ বৃত্ত এবং x = 3 রেখা দ্বারা সীমাবদ্ধ ক্ষুদ্রতম ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

(RUET. '04.05,D.B,14,CU.B.10,CTG.B,14,09, JB.13)

Solution: প্রদত্ত বৃত্তের সমীকরণ

$x^2 + y^2 = 25$ ……… (i)

বৃত্তের কেন্দ্র (0,0) এবং ব্যাসার্ধ $= 5$

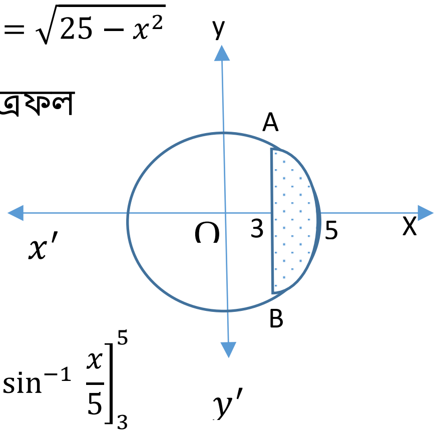
এবং রেখার সমীকরণ , $x = 3$ ……… (ii)

তাহলে, (ii) নং রেখা ও (i) নং বৃত্ত দ্বারা সীমাবদ্ধ ক্ষুদ্রতম ক্ষেত্রটির x এর সীমা 3 থেকে 5 পর্যন্ত এবং দুইটি চতুর্ভাগে সমান ভাগে বিভক্ত

(i) , নং থেকে আমরা পাই, $x^2 + y^2 = 25$

Or, $y^2 = 25 - x^2$

$\therefore y = \sqrt{25 - x^2}$



$\therefore$ নির্ণেয় ক্ষেত্রের ক্ষেত্রফল

$= 2\int_3^5 y\,dx$

$= 2\int_3^5 \sqrt{25 - x^2}\,dx$

$= 2\int_3^5 \sqrt{5^2 - x^2}\,dx$

$= 2\left[\frac{x\sqrt{5^2 - x^2}}{2} + \frac{5^2}{2}\sin^{-1}\frac{x}{5}\right]_3^5$

$\left[\because \int \sqrt{a^2 - x^2}\,dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$= \left(5\sqrt{5^2 - 5^2} + 25\sin^{-1}\frac{5}{5}\right) - \left(3\sqrt{5^2 - 3^2} + 25\sin^{-1}\frac{3}{5}\right)$

$= 0 + 25\sin^{-1} 1 - 3\sqrt{25 - 9} - 25\sin^{-1}\frac{3}{5}$

$= 25 \cdot \frac{\pi}{2} - 3.4 - 25\sin^{-1}\frac{3}{5}$

$= \left(\frac{25\pi}{2} - 25\sin^{-1}\frac{3}{5} - 12\right)$ বর্গ একক (Ans)

প্রশ্ন 13। . $x^2 + y^2 = 16$ বৃত্ত এবং x = 2 রেখা দ্বারা সীমাবদ্ধ ক্ষুদ্রতম ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

(S. B. ' 19)

Solution: প্রদত্ত বৃত্তের সমীকরণ.

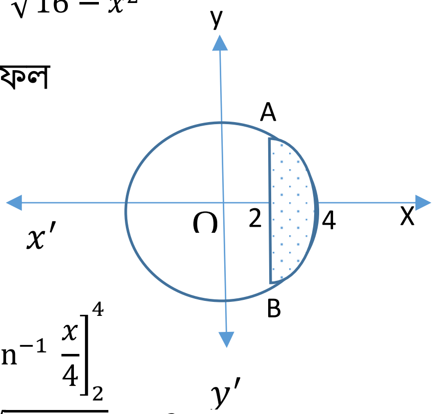
$x^2+y^2=16$ …… … (i)

বৃত্তের কেন্দ্র (0,0) এবং ব্যাসার্ধ $= 4$
এবং রেখার সমীকরণ , $x = 2$ ……… (ii)

∴তাহলে, (ii) নং রেখা ও (i) নং বৃত্ত দ্বারা সীমাবদ্ধ ক্ষুদ্রতম ক্ষেত্রটির x এর সীমা 2 থেকে 4পর্যন্ত এবং দুইটি চতুর্ভাগে সমান ভাগে বিভক্ত
(i) , নং থেকে আমরা পাই, $x^2+y^2=16$

Or, $y^2=16-x^2$
$\therefore y=\sqrt{16-x^2}$

∴ নির্ণেয় ক্ষেত্রের ক্ষেত্রফল



$= 2\int_2^4 y\,dx$
$= 2\int_2^4 \sqrt{16-x^2}\,dx$
$= 2\int_2^4 \sqrt{4^2-x^2}\,dx$
$= 2\left[\frac{x\sqrt{4^2-x^2}}{2}+\frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_2^4$
$\left[\because \int\sqrt{a^2-x^2}\,dx=\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$= \left(4\sqrt{4^2-4^2}+16\sin^{-1}\frac{4}{4}\right)-\left(2\sqrt{4^2-2^2}+16\sin^{-1}\frac{2}{4}\right)$
$= 0+16\sin^{-1}1-2\sqrt{16-4}-16\sin^{-1}\frac{2}{4}$
$= 16\cdot\frac{\pi}{2}-2.2\sqrt{3}-16\sin^{-1}\frac{2}{4}$
$= \left(8\pi-4\sqrt{3}-16\frac{\pi}{6}\right)$ বর্গ একক (Ans.)

প্রশ্ন 14। $x^2+y^2-8x=0$ বৃত্ত এবং $y=2x$ রেখা দ্বারা সীমাবদ্ধ ক্ষুদ্রতম ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।
[BB.19]
Solution:$y=2x$……………..(i)
$\mathbf{x^2+y^2-8x=0}$………. (ii)

$\mathbf{x^2-2.4.x+4^2-4^2+y^2=0}$

$\mathbf{x^2-2.4.x+4^2+y^2=4^2}$

$\mathbf{(x-4)^2+y^2=4^2}$

∴ বৃত্তের কেন্দ্র (4,0) এবং ব্যাসার্ধ $= 4$
$y=2x$ এর মান (ii) নং এ বসাই

or, $x^2+4x^2-8x=0$
or, $5x^2-8x=0$

or, $x(5x-8)=0$

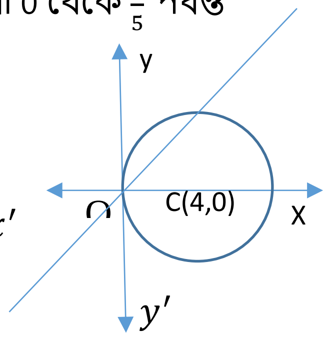
$\therefore x=0$ or $x=\frac{8}{5}$

তাহলে, (ii) নং রেখা ও (i) নং বৃত্ত দ্বারা সীমাবদ্ধ ক্ষুদ্রতম ক্ষেত্রটির x এর সীমা 0 থেকে $\frac{8}{5}$ পর্যন্ত

$x^2+y^2=8x$

or $y^2=8x-x^2$
$\therefore y=\sqrt{8x-x}$

∴ নির্ণেয় ক্ষেত্রের ক্ষেত্রফল



$= \int_0^{\frac{8}{5}}(y_1-y_2)dx = \int_0^{\frac{8}{5}}\left(2X-\sqrt{8x-x^2}\right)dx$

$= 2\int_0^{\frac{8}{5}}x\,dx-\int_0^{\frac{8}{5}}\sqrt{8x-x^2}\,dx$

এখন,

$\int\sqrt{8x-x^2}\,dx=\int\sqrt{16-16+8x-x^2}\,dx$
$= \int\sqrt{16-(x^2-8x+16)}\,dx$
$= \int\sqrt{16-(x-4)^2}\,dx$
$= \frac{(x-4)\sqrt{8x-x^2}}{2}+\frac{16}{2}\sin^{-1}\left(\frac{x-4}{4}\right)$
$= \frac{1}{2}(x-4)\sqrt{8x-x^2}+8\sin^{-}\left(\frac{x-4}{4}\right)$
$= \int_0^{\frac{8}{5}}\sqrt{8x-x^2}\,dx$

$= \left[\frac{1}{2}(x-4)\sqrt{8x-x^2}+8\sin^{-1}\left(\frac{x-4}{4}\right)\right]_0^{\frac{8}{5}}$
$= \frac{1}{2}\left(\frac{8}{5}-4\right)\sqrt{8\frac{8}{5}-\left(\frac{8}{5}\right)^2}+8\sin^{-1}\left(\frac{\frac{8}{5}-4}{4}\right)-\frac{1}{2}0-8\sin^{-1}\left(\frac{0-4}{4}\right)$

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$$=\frac{1}{2}\left(\frac{-12}{5}\right)\sqrt{\frac{320-64}{25}}+8\sin^{-1}\left(-\frac{3}{5}\right)-8\sin^{-1}(-1)$$

$$=-\frac{6}{5}\sqrt{\frac{256}{25}}+8\sin^{-1}\left(-\frac{3}{5}\right)+8\frac{\pi}{2}$$

$$=-\frac{6}{5}\cdot\frac{16}{5}+8\sin^{-1}\left(-\frac{3}{5}\right)+4\pi$$

$$=-\frac{96}{25}+8\sin^{-1}\left(-\frac{3}{5}\right)+4\pi$$

নির্ণেয়ক্ষেত্রফল

$$=2\int_0^5 xdx-\left[-\frac{96}{25}+8\sin^{-1}\left(-\frac{3}{5}\right)+4\pi\right]$$

$$=2\left[\frac{x^2}{2}\right]_0^{\frac{8}{5}}+\frac{96}{25}-8\sin^{-1}\left(-\frac{3}{5}\right)-4\pi$$

$$=\left(\frac{8}{5}\right)^2+\frac{96}{25}-8\sin^{-1}\left(-\frac{3}{5}\right)-4\pi$$

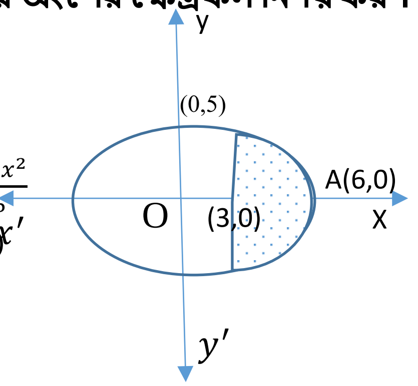
$$=\frac{64}{25}+\frac{96}{25}-8\sin^{-1}\left(-\frac{3}{5}\right)-4\pi$$

$$=\frac{160}{25}-8\sin^{-1}\left(-\frac{3}{5}\right)-4\pi$$

$$=\frac{32}{5}-8\sin^{-1}\left(-\frac{3}{5}\right)-4\pi$$

প্রশ্ন 15। $\frac{x^2}{36}+\frac{y^2}{25}=1$ উপবৃত্ত এবং $x=3$ সরল রেখা দ্বারা সীমাবদ্ধ ক্ষুদ্রতর অংশের ক্ষেত্রফল নির্ণয় কর।

[Raj.B 17]



Salution: $\frac{x^2}{36}+\frac{y^2}{25}=1$

or, $\frac{y^2}{25}=1-\frac{x^2}{36}=\frac{36-x^2}{36}$

or. $y^2=\frac{25}{36}(36-x^2)$

$y=\frac{5}{6}\sqrt{36-x^2}$

আবার,

, $\frac{x^2}{36}+\frac{y^2}{25}=1$

or, $\frac{x^2}{6^2}+\frac{y^2}{5^2}=1$..................(i)

(i) নং উপবৃত্তটি x ও y অক্ষকে $(\pm6,0)$ এবং $(0,\pm5)$ বিন্দুতে ছেদ করে এবং $x=3$ রেখা ও (i) নং উপবৃত্তটি দ্বারা সীমাবদ্ধ ক্ষুদ্রতম ক্ষেত্রটির x এর সীমা 3 থেকে 6 পর্যন্ত ।

∴ নির্ণেয় ক্ষেত্রফল

$$=2\int_3^6\frac{5}{6}\sqrt{36-x^2}dx=\frac{5}{6}\cdot2\int_3^6\sqrt{6^2-x^2}dx$$

$$=\frac{5}{3}\left[\frac{x\sqrt{36-x^2}}{2}+\frac{36}{2}\sin^{-1}\frac{x}{6}\right]_3^6$$

$$=\frac{5}{3}\left[\left(0+18\sin^{-1}(1)-\frac{3}{2}\cdot\sqrt{27}+18\sin^{-1}\left(\frac{1}{2}\right)\right]\right.$$

$$=\frac{5}{3}\left\{\left(0+18\frac{\pi}{2}\right)-\frac{3}{2}\cdot3\sqrt{3}-18\cdot\frac{\pi}{6}\right\}$$

$$=\frac{5}{3}\left(9\pi-\frac{9\sqrt{3}}{2}-3\pi\right)$$

$$=\frac{5}{3}\left(6\pi-\frac{9\sqrt{3}}{2}\right)$$

$$=5\left(2\pi-\frac{3\sqrt{3}}{2}\right)$$ বর্গ একক (Ans.)

প্রশ্ন 16। $9x^2+16y^2-144=0$ উপবৃত্ত এবং $x-2=0$ সরল রেখা দ্বারা সীমাবদ্ধ ক্ষুদ্রতর অংশের ক্ষেত্রফল নির্ণয় কর।[S. B. '17]

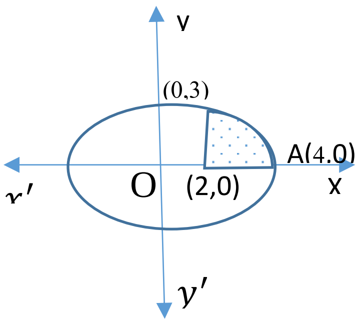
Solution $9x^2+16y^2-144=0$

$\Rightarrow 9x^2+16y^2=144$

$\Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$

$\Rightarrow \frac{x^2}{4^2}+\frac{y^2}{3^2}=1$

এবং $x-2=0\therefore x=2$



আবার,

$\Rightarrow y=\sqrt{\frac{9}{16}(16-x^2)}$

$\therefore y=\frac{3}{4}\sqrt{16-x^2}$

উপবৃত্তটি x-অক্ষকে $(\pm4,,0)$ এবং y-অক্ষকে $(0,\pm3)$ বিন্দুতে ছেদ করে। উপবৃত্ত এবং $x=2$ রেখার মধ্যবর্তী অংশের ক্ষেত্রফল ।

∴ নির্ণেয় ক্ষেত্রফল $= 2\int_2^4 ydx$

$$= 2\int_2^4 \frac{3}{4}\sqrt{16-x^2}dx$$

$$= 2\int_2^4 \frac{3}{4}\sqrt{16-x^2}dx = \frac{3}{4}\cdot 2\int_2^4 \sqrt{16-x^2}dx$$

$$= \frac{3}{2}\left[\frac{x\sqrt{16-x^2}}{2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$$

$$= \frac{3}{2}\left[\left(0+8\sin^{-1}(1)-\sqrt{12}-8\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$$

$$= \frac{3}{2}\left\{\left(0+8\frac{\pi}{2}\right)-2\sqrt{3}-8\cdot\frac{\pi}{6}\right\}$$

$$= \frac{3}{2}\left(4\pi-2\sqrt{3}-4\cdot\frac{\pi}{3}\right)$$

$$= \frac{3}{2}\left(\frac{8\pi}{3}-2\sqrt{3}\right)$$ *বর্গ একক* ।

প্রশ্ন 17। $y^2 = 16x$ পরাবৃত্ত এবং এর উপকেন্দ্রিক লম্ব দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। [S.B 05]

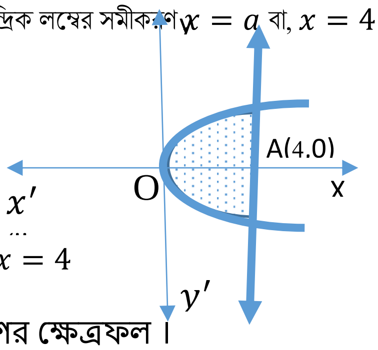
Solution: $y^2 = 16x = 4.4\cdot x$ সমীকরণকে $y^2 = 4ax$

এর সাথে তুলনা করে পাই $a = 4$

$y^2 = 4ax$ এর উপকেন্দ্রিক লম্বের সমীকরণ $x = a$ বা, $x = 4$

আবার ,. $y^2 = 16x$

$\Rightarrow y = 4\sqrt{x}$



প্রদত্ত পরাবৃত্ত এবং $x = 4$

রেখার মধ্যবর্তী অংশের ক্ষেত্রফল ।

∴ নির্ণেয় ক্ষেত্রফল $= 2\ [x = 0, x = 4, y^2 = 16x$ এবং, x অক্ষ দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল]

$$= 2\int_0^4 ydx = 2\int_0^1 4\sqrt{x}dx$$

$$= 2.4\int_0^4 x^{\frac{1}{2}}dx = 8\cdot\frac{2}{3}\left[x^{\frac{1}{2}}\right]_0^4 = \frac{16}{3}\left[(4)^{\frac{1}{2}}-(0)^{\frac{1}{2}}\right]$$

$$= \frac{16}{3}(8-0) = \frac{128}{3}$$ বর্গ একক ।(ans)

প্রশ্ন 18। $y^2 = 16x$ পরাবৃত্ত এবং y= x সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। [KUET.11-12; D.03,S.02]

Solution: দেওয়া আছে, $y^2 = 16x$ ....(i)

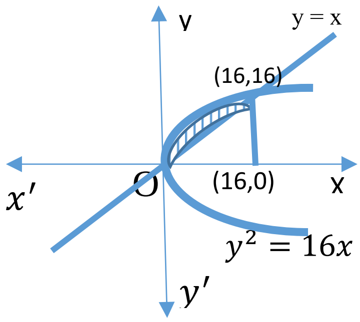
এবং $y = x$..............................(ii)

(i)ও (ii) সমাধান করে পাই, $x^2 = 16x$

$x^2 - 16x = 0$

x(x-16)=0

∴ $x = 0, x = 16$



আবার, (i) *নং থেকে পাই* $y = 4\sqrt{x} = y_1$ (ধরি)

(ii)নং থেকে পাই $y = x = y_2$ (ধরি)

(i)নং পরাবৃত্ত এবং (ii)নং রেখার মধ্যবর্তী অংশের ক্ষেত্রফল ।

$$= \int_0^{16}(y_1 - y_2)dx$$

$$= \int_0^{16}(4\sqrt{x} - x)dx$$

$$= \frac{8}{3}\left[x^{\frac{3}{2}}\right]_0^{16} - \left[\frac{x^2}{2}\right]_0^{16}$$

$$= \frac{8}{3}\cdot(16)^{\frac{3}{2}} - \frac{1}{2}\cdot(16)^2$$

$$= \frac{8}{3}\cdot 64 - \frac{1}{2}\cdot 256 = \frac{512}{3} - 128$$

$$= \frac{512-384}{3} = \frac{128}{3}$$ বর্গ একক ।(ans)

প্রশ্ন 19। $y^2 = 7x$ পরাবৃত্ত এবং y= x সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। [C.B 19]

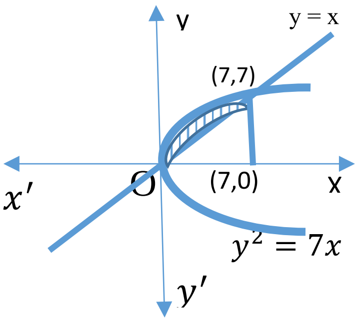
Solution: দেওয়া আছে, $y^2 = 7x$ ....(i)

এবং $y = x$..............................(ii) (i)ও (ii) সমাধান করে পাই , $x^2 = 7x$

$x^2 - 7x = 0$

x(x-7)=0

$\therefore x = 0, x = 7$



আবার, (i) নং থেকে পাই $y = \sqrt{7x} = y_1$ (ধরি)

(ii)নং থেকে পাই $y = x = y_2$ (ধরি)

(i)নং পরাবৃত্ত এবং (ii)নং রেখার মধ্যবর্তী অংশের ক্ষেত্রফল ।

$= \int_0^7 (y_1 - y_2)dx$

$= \int_0^7 (7x)^{\frac{1}{2}}dx - \int_0^7 xdx$

$= 7^{\frac{1}{2}}\left[\frac{x^{\frac{2}{2}}}{\frac{3}{2}}\right]_0^7 - \left[\frac{x^2}{2}\right]_0^7$

$= \frac{2}{3}7^{\frac{1}{2}}\left[7^{\frac{3}{2}} - 0\right] - \frac{1}{2}[7^2 - 0]$

$= \frac{2}{3}7^{\frac{1}{2}+\frac{3}{2}} - \frac{1}{2}(49) = \frac{2}{3}(49) - \frac{1}{2}(49)$

$= 49\left(\frac{2}{3} - \frac{1}{2}\right) = 49\left(\frac{4-3}{6}\right)$

$= \frac{49}{6}$ বর্গ একক ।(ans)

প্রশ্ন 20। $y^2 = 6x$ পরাবৃত্ত এবং y= x সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। [R.B 19]

Solution: দেওয়া আছে , $y^2 = 6x$ ....(i)

এবং $y = x$................................(ii)

(i)ও (ii) সমাধান করে পাই , $x^2 = 6x$

$x^2 - 6x = 0$

x(x-6)=0

$\therefore x = 0, x = 6$

আবার, (i) নং থেকে পাই $y = \sqrt{6x} = y_1$ (ধরি)

(ii)নং থেকে পাই $y = x = y_2$ (ধরি)

(i)নং পরাবৃত্ত এবং (ii)নং রেখার মধ্যবর্তী অংশের ক্ষেত্রফল ।

$= \int_0^6 (y_1 - y_2)dx$

$= \int_0^6 (6x)^{\frac{1}{2}}dx - \int_0^6 xdx$

$= 6^{\frac{1}{2}}\left[\frac{x^{\frac{2}{2}}}{\frac{3}{2}}\right]_0^6 - \left[\frac{x^2}{2}\right]_0^6$

$= \frac{2}{3}6^{\frac{1}{2}}\left[6^{\frac{3}{2}} - 0\right] - \frac{1}{2}[6^2 - 0]$

$= \frac{2}{3}6^{\frac{1}{2}+\frac{3}{2}} - \frac{1}{2}(36) = \frac{2}{3}(36) - \frac{1}{2}(36)$

$= 36\left(\frac{2}{3} - \frac{1}{2}\right) = 36\left(\frac{4-3}{6}\right)$

$= \frac{36}{6} = 6$ বর্গ একক ।(ans)

প্রশ্ন 21। $y = 4x^2$ পরাবৃত্ত এবং y= 4সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। [C.B. '01]

Solution: $y = 4x^2$ পরাবৃত্তের শীর্ষ বিন্দু $O(0,0)$

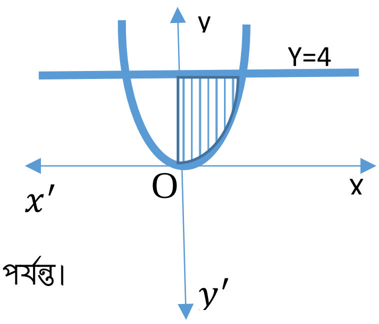
$$y = 4x^2 \Rightarrow x^2 = \frac{1}{4}y \Rightarrow x = \frac{1}{2}\sqrt{y}$$

$y = 4x^2$ পরাবৃত্ত

এবং y = 4 সরলরেখা

দ্বারা সীমাবদ্ধ ক্ষেত্রে

y-এর সীমা 0 থেকে 4 পর্যন্ত।



∴ ক্ষেত্র OAB এর ক্ষেত্রফল =

[y = 4x বক্ররেখা, y অক্ষ এবং y = 0 ও y = 4 রেখাদ্বয় দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল]

$= \int_0^4 xdy = \frac{1}{2}\int_0^4 \sqrt{y}dy$

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$$= \frac{1}{2}\left[\frac{y^{\frac{1}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{1}{2}\times\frac{2}{3}\left\{(4)^{\frac{3}{2}} - 0\right\}$$

$$= \frac{1}{3}\times 8 = \frac{8}{3}$$ বর্গ একক ।(ans)

∴নির্ণেয় আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= 2 \times$

OABক্ষেত্রের ক্ষেত্রফল $= \frac{16}{3}$ বর্গ একক ।(ans)

প্রশ্ন 22। $y^2 = 4x$ পরাবৃত্ত এবং y= x সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।.[CUET 13-14,BUTEX '12-13'; $D, 13$, $CU. B\ '15$]

Solution: দেওয়া আছে, $y^2 = 4x$ ....(i)

এবং $y = x$................................(ii)

(i)ও (ii) সমাধান করে পাই, $x^2 = 4x$

$x^2 - 4x = 0$

x(x-4)=0

$\therefore x = 0, x = 4$

আবার, (i) *নং থেকে পাই* $y = 2\sqrt{x} = y_1$(ধরি)

(ii)নং থেকে পাই $y = x = y_2$ (ধরি)

(i)নং পরাবৃত্ত এবং (ii)নং রেখার মধ্যবর্তী অংশের ক্ষেত্রফল ।

$$= \int_0^4 (y_1 - y_2)dx$$

$$= \int_0^4 (2\sqrt{x} - x)dx$$

$$= \int_0^4 2\sqrt{x}dx - \int_0^4 xdx$$

$$= 2\int_0^4 \sqrt{x}dx - \left[\frac{x^2}{2}\right]_0^4$$

$$= 2\left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^4 - \left[\frac{x^2}{2}\right]_0^4$$

$$= 2\times\frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^4 - \frac{1}{2}(4^2 - 0)$$

$$= 2\cdot\frac{2}{3}\left[4^{\frac{3}{2}} - 0\right] - \frac{1}{2}\times 16$$

$$= \frac{4}{3}\cdot 8 - 8 = \frac{32}{3} - 8 = \frac{32-24}{3}$$

$$= \frac{8}{3}$$ বর্গ একক ।(ans)

প্রশ্ন 23। $y^2 = x$ পরাবৃত্ত এবং y= x -2 সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।. [BUET.10-11,

Solution: এখানে, $y^2 = x$ ....…….(i)

এবং, $y = x - 2$.................(ii)

(i) নং থেকে পাই, $(x-2)^2 = x$

$\Rightarrow x^2 - 4x + 4 = x \Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow (x-1)(x-4) = 0 \therefore x = 1,4$

(ii) নং এ x এর মান বসাই,

যখন, $x = 1$, তখন, $y = 1 - 2 = -1$

যখন, $x = 4$, তখন, $y = 4 - 2 = 2$

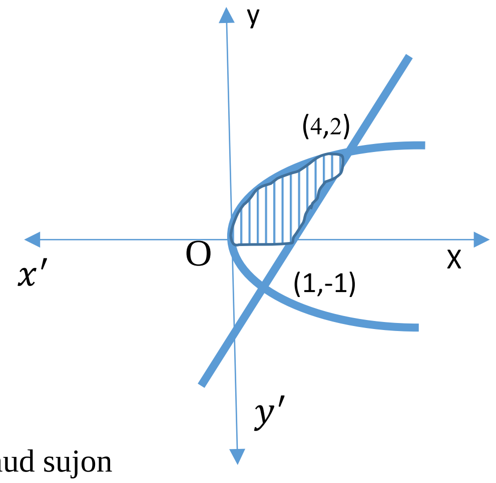
∴ ছেদ বিন্দুর স্থানাঙ্ক , $(1,-1), (4,2)$

নির্ণেয় আবদ্ধ ক্ষেত্রে y এর সীমা -1 থেকে 2 পর্যন্ত।

(ii) $\Rightarrow x = y + 2 = x_1$ (ধরি)

(i) $\Rightarrow x = y^2 = x_2$ (ধরি)

∴ নির্ণেয় আবদ্ধ ক্ষেত্রের ক্ষেত্রফল



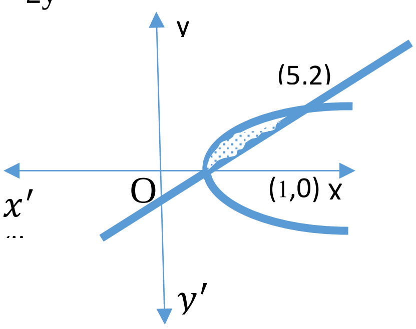
$$= \int_{-1}^{2} (x_1 - x_2)dy$$
$$= \int_{-1}^{2} \{(y+2) - y^2\}dy$$
$$= \int_{-1}^{2} ydy + 2\int_{-1}^{2} dy - \int_{-1}^{2} y^2dy$$
$$= \left[\frac{y^2}{2}\right]_{-1}^{2} + 2[y]_{-1}^{2} - \left[\frac{y^3}{3}\right]_{-1}^{2}$$
$$= \frac{1}{2}[2^2 - (-1)^2] + 2[2-(-1)] - \frac{1}{3}[2^3 - (-1)^3]$$
$$= \frac{1}{2}(4-1) + 2(2+1) - \frac{1}{3}(8+1)$$
$$= \frac{1}{2}\cdot 3 + 6 - 3 = \frac{3}{2} + 3$$
$= \frac{3+6}{2} = \frac{9}{2}$ বর্গ একক । (ans)

প্রশ্ন 24। $y^2 = x$ পরাবৃত্ত এবং y= x -2 সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।. [BUET.14-15]

Solution: এখানে, $y^2 = x - 1$ ... ... (i)

$2y = x - 1$ ... ... (ii)

(i)ও (ii) নং থেকে পাই, $y^2 = 2y$



$y^2 - 2y = 0$

y (y -2) = 0

$\therefore y = 0, y = 2$

$y = 0$ হলে x= 1

$y = 2$ হলে x= 5

∴ ছেদ বিন্দুর স্থানাঙ্ক , (1,0), (5,2)

∴ নির্ণেয় আবদ্ধ ক্ষেত্রে xএর সীমা 1 থেকে 5 পর্যন্ত।

(i) $\Rightarrow y = \sqrt{x-1} = y_1$ (ধরি)

(ii) $\Rightarrow y = \frac{x}{2} - \frac{1}{2} = y_2$ (ধরি)

∴ নির্ণেয়ক্ষেত্রফল $= \int_{1}^{5} (y_1 - y_2)dx$

$$= \int_{1}^{5} \left\{\sqrt{x-1} - \left(\frac{x}{2} - \frac{1}{2}\right)\right\} dx$$
$$= \int_{1}^{5} \left\{(x-1)^{\frac{1}{2}} - \frac{x}{2} + \frac{1}{2}\right\} dx$$
$$= \left[\frac{(x-1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{4} + \frac{1}{2}x\right]_{1}^{5}$$
$$-\frac{2}{3}(5-1)^2 - \frac{4^1}{4} + \frac{5}{2} - \left\{\frac{2}{3}(1-1)^3 - \frac{1}{4}, \frac{1}{2}\right\}$$
$$\left(\frac{10}{3} - \frac{25}{4} + \frac{5}{3}\right) + \left(0 - \frac{1}{4} + \frac{1}{2}\right)$$
$$-\frac{16}{3} - \frac{25}{4} + \frac{5}{2} + \frac{1}{4} - \frac{1}{2}$$
$$\frac{64 - 75 + 30 + 3 - 0}{12}$$
$= \frac{16}{12} = \frac{4}{3}$ বর্গ একক । (ans)

প্রশ্ন 25।দেখাও যে, x^2 = y পরাবৃত্ত এবং x – y = 0 সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল $\frac{1}{6}$ ।

[Raj.Ctg.Cu. 18]

Solution: দেওয়া আছে, $x^2 = y$ ....(i)

$x - y = 0$ ... ... (ii)

(i)ও (ii) নং থেকে পাই, $x^2 = x$

or, $x^2 - x = 0$

or, $x(x-1) = 0$

or, $x = 0,1$

$\therefore x = 0$ হলে $y = 0$এবং $x = 1$ হলে, $y = 1$

∴ ছেদ বিন্দুর স্থানাঙ্ক , (0,0)এবং, (1,1).

∴ নির্ণেয় আবদ্ধ ক্ষেত্রে xএর সীমা 0 থেকে 1 পর্যন্ত।

$\therefore$ নির্ণেয়ক্ষেত্রফল $= \int_0^1 (x - x^2)dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= \left(\frac{1}{2} - \frac{1}{3}\right) - (0 - 0)$

$= \frac{3-2}{6} = \frac{1}{6}$ বর্গ একক। (ans)

প্রশ্ন 26।দেখাও যে, $x^2 = y$ পরাবৃত্ত এবং $y = x + 6$ সরলরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল

[D.B.S.B.J.B. 18]

Solution: দেওয়া আছে, $y = x + 6$ ... (i) এবং

$x^2 = y$ .... (ii)

$\therefore x^2 = x + 6$

or, $x^2 - x - 6 = 0$

or, $x^2 - 3x + 2x - 6 = 0$

or,(x-3)(x+2)=0

$\therefore x = 3, -2$

$\therefore$ ছেদ বিন্দুর স্থানাঙ্ক , $(-2,4)$ এবং $(3,9)$

$\therefore A = \int_{-2}^{3} (y_1 - y_2)dx = \int_{-2}^{3} (x + 6 - x^2)dx$

$= \left[\frac{x^2}{2} + 6x - \frac{x^3}{3}\right]_{-2}^{3} = \left(\frac{9}{2} + 18 - \frac{27}{3}\right) - \left(\frac{4}{2} - 12 + \frac{8}{3}\right)$

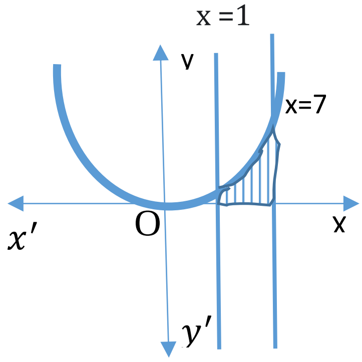
$= \frac{9}{2} + 18 - 9 - 2 + 12 - \frac{8}{3} = \frac{9}{2} - \frac{8}{3} + 19$

$= \frac{27 - 16 + 114}{6} = \frac{125}{6}$ বর্গ একক। (ans.)

প্রশ্ন 27। $y = x^2$ পরাবৃত্ত x-অক্ষ এবং x = 1 ও y = 7 রেখাদ্বয় সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। [C.B 02]

Solution: $y = x^2$ ....(i)

(i)নং বক্ররেখা X অক্ষ এবং x = 1 ও X = 7 রেখাদ্বয় দ্বারা আবদ্ধ ক্ষেত্রে X এর সীমা 1 থেকে 7 পর্যন্ত ।



$\therefore$ নির্ণেয় আবদ্ধ ক্ষেত্রের ক্ষেত্রফল = [y = x বক্ররেখা, x অক্ষ এবং x = 1 ও x = 7 রেখাদ্বয় দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল]

$= \int_1^7 ydx = \int_1^7 x^2dx = \left[\frac{x^3}{3}\right]_1^7$

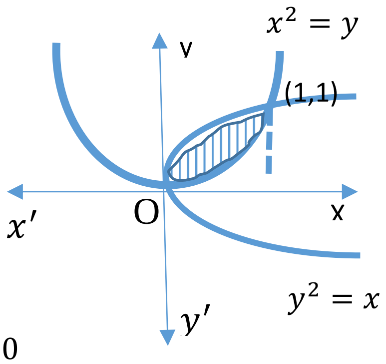
$= \frac{1}{3}(343 - 1) = 114$ বর্গ একক। (ans)

প্রশ্ন 28। $y^2 = x$ এবং $x^2 = y$ বক্ররেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। [J.'10: BU TEX.05-06]

Solution: দেওয়া আছে, $y^2 = x$..................(i)

এবং $x^2 = y$ ... ... (ii)

(i)ও (ii) নং থেকে পাই,



$\therefore (x^2)^2 = x$

$\Rightarrow x(x^3 - 1) = 0$

$\therefore x = 0,1$

যখন, $x = 0$, (ii) $\Rightarrow y = 0$

যখন, $x = 1$, (ii) $\Rightarrow y = 1$ (1,1)

তাহলে, (i) ও (ii) নং পরাবৃত্তদ্বয় দ্বারা আবদ্ধ ক্ষেত্রে x এর সীমা 0 থেকে 1 পর্যন্ত।

(i)নং থেকে পাই, $y = \sqrt{x} = y_1$ (ধরি)

(ii) নং থেকে পাই, $y = x^2 = y_2$ (ধরি)

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$\therefore$ নির্ণেয় ক্ষেত্রফল $= \int_0^1 (y_1 - y_2)\mathrm{d}x$

$= \int_0^1 (\sqrt{x} - x^2)dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right]_0^1$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ বর্গ একক। (ans.)

প্রশ্ন 29। $x^2 + y^2 = 1$ এবং $y^2 = 1 - x$ দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর। [D.B, '01]

Solution দেওয়া আছে বৃত্তের সমীকরণ, $x^2 + y^2 = 1$ ………(i)

$y^2 = 1 - x^2$

$\therefore y = \sqrt{1 - x^2} = y_1$ (ধরি)

পরাবৃত্তের সমীকরণ, y^2 = 1-x……………………………(ii)

$\therefore y = \sqrt{1 - x} = y_2$ (ধরি)

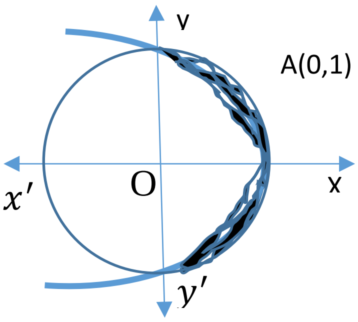
(i) নং থেকে (ii) বিয়োগ করি ,

$x^2 = x$

Or, $x^2 - x = 0$

x (x-1) = 0

$\therefore$ x = 0 ,1



যখন $x = 0$, তখন (ii) $\Rightarrow y = \pm 1$

যখন, $x = 1$, তখন (ii) $\Rightarrow y = 0$

$\therefore$ বৃত্ত *এবং* পরাবৃত্ত *অক্ষকে* $(1,0), (0,1)$ এবং$(0 - 1)$. বিন্দুতে ছেদ করে

$\therefore$ নির্ণেয় ক্ষেত্রফল $= 2\int_0 (y_1 - y_2)dx$

$= 2\int_0^1 \left(\sqrt{1 - x^2} - \sqrt{1 - x}\right) dx$

$= 2\left\{\int_0^1 \sqrt{1 - x^2}dx - \int_0^1 \sqrt{1 - x}dx\right\}$

এখন , $\int_0^1 \sqrt{1 - x^2}dx$

$= \int_0^{\pi/2} \cos\theta \cdot \cos\theta d\theta$

ধরি, $x = \sin\theta$

$\therefore dx = \cos\theta d\theta$

x	1	θ
θ	$\pi/2$	0

$= \frac{1}{2}\int_0^{\pi/2} 2\cos^2\theta d\theta$

$= \frac{1}{2}\int_0^{\pi/2} (1 + \cos 2\theta)d\theta$

$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2 0\right]_0^{\pi 2}$

$= \frac{1}{2}\left[\frac{\pi}{2} + \frac{1}{2}\sin x - \left(0 - \frac{1}{2}\sin\theta\right)\right]$

$= \frac{1}{2}\left(\frac{\pi}{2} + 0 - 0\right) = \frac{\pi}{4}$

আবার , $\int_0^1 \sqrt{1 - x}dx = -\frac{2}{3}\left[(1 - x)^{\frac{3}{2}}\right]_0^1$

$=: -\frac{2}{3}\left[(1 - 1)^{\frac{3}{2}} - (1 - 0)^{\frac{3}{2}}\right]$

$= -\frac{2}{3}(0 - 1) = \frac{2}{3}$

$\therefore$ নির্ণেয় ক্ষেত্রফল $= 2\left(\frac{\pi}{4} - \frac{2}{3}\right)$ বর্গ একক। (ans.)

প্রশ্ন 30। $xy = c^2$, x – অক্ষ এবং $x = a, x = b(b > a > 0)$দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

Solution: দেওয়া আছে, বক্র রেখের সমীকরণ

$xy = c^2$

$\therefore \quad y = \frac{c^2}{x}$ ……..(i)

(i)নং অধি বৃত্ত $x = \mathrm{a}$ *এবং* $x = b$.রেখা দ্বারা সীমাবদ্ধ

$\therefore$ নির্ণেয় ক্ষেত্রফল, $= \int_{\mathrm{a}}^{\mathrm{b}} y\mathrm{d}x$

$= \int_a^b \frac{c^2}{x}\mathrm{d}x$

$= c^2\ [\ln x]_{\mathrm{a}}^{\mathrm{b}}$

$= c^2[\ln b - \ln a]$

$= c^2 \ln\left(\frac{b}{a}\right)$ বর্গ একক। (ans.)

প্রশ্ন 31। $y = \cos x$ এই বক্ররেখা দ্বারা x অক্ষের একটি আবদ্ধ চাপের ক্ষেত্রফল নির্ণয় কর। [C. B.19]

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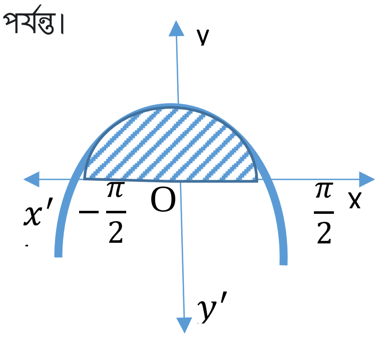
Solution: $y = \cos x$ এই বক্ররেখা দ্বারা x অক্ষের একটি চাপের সীমা $-\frac{\pi}{2}$ থেকে $\frac{\pi}{2}$ থেকে পর্যন্ত।



∴ নির্ণেয় ক্ষেত্রফল

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} y dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = [\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)$$

$$= \sin\frac{\pi}{2} + \sin\frac{\pi}{2}$$

$$= 2\sin\frac{\pi}{2}$$

$$= 2.1$$

$= 2$ বর্গ একক। (ans.)

অধ্যায়-১০: যোগজীকরণ (১ম পত্র)

❖ $\mathbf{I = \int_0^{\frac{\pi}{2}} \sin^5 x\, dx}$

$$= \left[-\cos x + 2.\frac{\cos^3 x}{2} - \frac{\cos^5 x}{5}\right]_0^{\frac{\pi}{2}}$$

$$= 0 - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)$$

$$= 1 - \frac{2}{3} + \frac{1}{5}$$

$$= \frac{15-10+3}{15}$$

$$= \frac{8}{15}$$

❖ $\mathbf{I = \int_0^{\frac{\pi}{2}} \cos^5 x\, dx}$.

$$= \left[\sin x - 2\frac{\sin^3 x}{3} + \frac{\sin^5 x}{5}\right]_0^{\frac{\pi}{2}}$$

$$= \left(1 - \frac{2}{3} + \frac{1}{5}\right) - 0$$

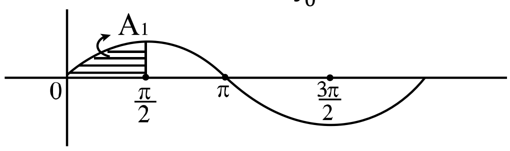
$$= \frac{8}{15}$$

সুতরাং, $\boxed{\int_0^{\frac{\pi}{2}} \sin^5 x\, dx = \int_0^{\frac{\pi}{2}} cos^5 x\, dx}$

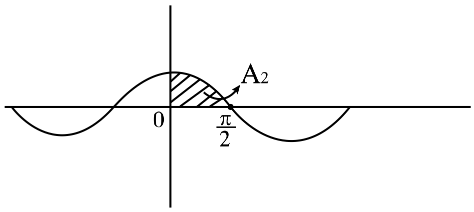
এখন,

- $\int_0^{\frac{\pi}{2}} cos^5 x\, dx = \frac{4.2}{5.3.1} = \frac{8}{15}$ [2 করে কমবে]
- $\int_0^{\frac{\pi}{2}} \cos^7 x\, dx = \frac{6.4.2}{7.5.3.1} = \frac{2.4.2}{7.5.1} = \frac{16}{35}$
- $\int_0^{\frac{\pi}{2}} \sin^9 x\, dx = \frac{8.6.4.2}{9.7.5.3.1}$ [Wall's theorem]
- $\int_0^{\frac{\pi}{2}} \cos^{11} x\, dx = \frac{10.8.6.4.2}{11.9.7.5.3.1}$
- $\boxed{\int_0^{\frac{\pi}{2}} \sin x\, dx = \int_0^{\frac{\pi}{2}} cos\, x\, dx}$

$$\int_0^{\frac{\pi}{2}} sin\, x\, dx = A_1$$



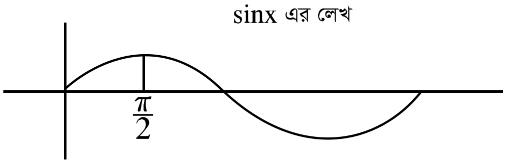
$$\int_0^{\frac{\pi}{2}} cos\, x\, dx = A_2$$



অর্থাৎ, $A_1 = A_2$

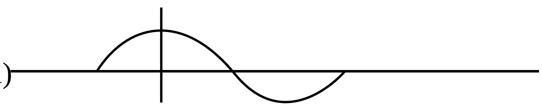
graph transfer

sinx এর লেখ (i)



$\frac{\pi}{2}$ কে 0 – এ টেনে পেছনে আনলে মূলত $cosx$ লেখ পাওয়া যায়।

(ii)



আবার, (i)কে $sinx$ বললে (ii)কে বলা হবে $\sin\left(\frac{\pi}{2} - x\right)$ লেখ। মানে হলো, x এর স্থলে $\left(\frac{\pi}{2} - x\right)$ বসাতে হবে।

$$\therefore \int_0^{\frac{\pi}{2}} sin\, x\, dx = \int_0^{\frac{\pi}{2}} cos\, x\, dx$$
$$= \int_0^{\frac{\pi}{2}} \sin\left(\frac{\pi}{2} - x\right) dx$$

❖ **সারমর্ম:**

$$\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$$

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$$\int_a^b f(x)\ dx = \int_a^b f(t)\,dt$$

$$\int_a^b f(x)\ dx = -\int_b^a f(x)\,dx$$

প্রমাণ:

$I = \int_0^a f(a-x)\,dx$

$I = -\int_a^o f(z)\,dz$

$= \int_o^a f(z)\,dz$

$= \int_o^a f(x)\,dx$

$a - x = z$

$\Longrightarrow\ -dx = dz.$

$\Longrightarrow\ dx = -dz.$

x	o	a
z	a	o

❖ $\int_a^b \boldsymbol{f(x)\,dx} = \int_a^b \boldsymbol{f(a+b-x)\,dx}$

প্রমাণ: $I = \int_a^b f(a+b-x)\,dx$

$= -\int_b^a f(z)\,dz$

$= \int_a^b f(z)\,dz\ = \int_a^b f(x)\,dx$

$a + b - x = z$

$\therefore\ -dx = dz.$

$\therefore\ dx = -dz.$

x	a	b
z	b	a

$Type - 01$:

- $\int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x+\cos^5 x}dx$
- $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x+\cos^5 x}dx$
- $\int_0^{\frac{\pi}{2}} \frac{\tan^5 x}{\tan^5 x+\cot^5 x}dx$
- $\int_0^{\frac{\pi}{2}} \frac{\cot^5 x}{\tan^5 x+\cot^5 x}dx$
- $\int_0^{\frac{\pi}{2}} \frac{\sec^5 x}{\sec^5 x+\text{cosec}^5 x}dx$
- $\int_0^{\frac{\pi}{2}} \frac{\text{cosec}^5 x}{\sec^5 x+\text{cosec}^5 x}dx$

ধরি, $I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x+\cos^5 x}dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$

$I = \int_0^{\frac{\pi}{2}} \frac{\sin^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)\ +\cos^5\left(\frac{\pi}{2}-x\right)}dx$

$\Longrightarrow\ I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x+\ cos^5 x}dx \ldots\ldots\ldots\ldots..(ii)$

$(i) + (ii),$

$2I\ = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x+\cos^5 x}{sin^5 x+\cos^5 x}dx$

$= \int_0^{\frac{\pi}{2}} dx$

$= [x]_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2}$

$\therefore I = \frac{\pi}{4}$ **(Ans)**

❖ $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^5 x}{\sin^5 x+\cos^5 x}dx \ldots\ldots\ldots\ldots\ldots\ldots.(i)$

$\Longrightarrow\ I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)}dx$ $\left[\because \int_a^b f(x)dx = \int_a^b f\,(a+b-x)dx\right]$

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$\Longrightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos^5 x\, dx}{sin^5 x + \cos^5 x} dx \ldots\ldots\ldots\ldots..(ii)$

$(i) + (ii),$

$2\mathrm{I} = \int_{\pi/6}^{\pi/3} dx$

$= [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

$\Longrightarrow I = \frac{\pi}{12}$

Note: অর্থাৎ, **Lower limit +Upper limit = $\frac{\pi}{2}$ হলে, $I = \frac{Upper\ limit - Lower\ limt}{2}$**

❖ $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\tan^{100} x}{tan^{100} x + \cot^{100} x} dx = \frac{\frac{\pi}{3} - \frac{\pi}{6}}{2} = \frac{\pi}{12}$

❖ $I = \int_0^{\frac{\pi}{2}} \frac{x \sin x . cosx}{sin^4 x + \cos^4 x} dx$

$= \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right) dx}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)}$

$\left[\therefore \int_0^a f(x)\ dx = \int_0^a f(a - x)\, dx\right]$

$= \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) cosx\ sinx\ \ dx}{\cos^4 x + \sin^4 x}$

$\therefore 2\mathrm{I} = \int_0^{\frac{\pi}{2}} \frac{\left(x + \frac{\pi}{2} - x\right) sinx . cosx}{\sin^4 x + \cos^4 x} dx.$

$\Longrightarrow 2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{sinx . cosx}{\sin^4 x + \cos^4 x} dx.$

$\Longrightarrow I = \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \frac{tanx . \sec^2 x}{\tan^4 x + 1} dx.$ [লব ও হরকে $\cos^4 x$ দ্বারা ভাগ]

$\Longrightarrow I = \frac{1}{2} \frac{\pi}{4} \int_0^{\infty} \frac{dz}{1 + z^2}$

$= \frac{1}{2} \frac{\pi}{4} [\tan^{-1} z]_0^{\infty}$

$= \frac{\pi}{8} \left[\frac{\pi}{2} - 0\right] = \frac{\pi^2}{16}.$

$z = \tan^2 x$

$dz = 2tanx \sec^2 x\, dx$

x	0	$\pi/2$
z	0	∞

❖ $\boldsymbol{I = \int_o^4 y \sqrt{4 - y}\, dy}.$

$= \int_0^4 (4 - y) \sqrt{y} dy.$

$= \int_0^4 (4y^{\frac{1}{2}} - y^{\frac{3}{2}})\, dy$

$= \left[4 . \frac{y^{\frac{3}{2}}}{\frac{3}{2}} - \frac{y^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^4$

$= \left[\frac{8}{3} \times 4\sqrt{4} - \frac{2}{5} . 4^2\sqrt{4}\right] - 0$

$= \frac{8\times8}{3} - \frac{2\times32}{5}$

$= \frac{64}{3} - \frac{64}{5}$

$= \frac{2\times64}{15}$

$= \frac{128}{15}$

❖ $\boldsymbol{I = \int_0^1 x\,(1 - x)^n dx}$

$$= \int_0^1 (1-x)\{1-(1-x)\}^n dx$$

$$= \int_0^1 (1-x)\, x^n dx \qquad \left[\because \int_0^a f(x)\, dx = \int_0^a f(a-x) dx\right]$$

$$= \int_0^1 (x^n - x^{n+1}) dx.$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \frac{1}{n+1} - \frac{1}{n+2}$$

$$= \frac{n+2-n-1}{(n+1)\,(n+2)} = \frac{1}{(n+1)(n+2)}$$

★★★ আমরা $\int \frac{dx}{1+sinx}$ এর সমাধান পারি।

কিন্তু, $I = \int_{\pi/4}^{3\pi/4} \frac{x}{1+sinx} dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$

$$\Rightarrow I = \int_{\pi/4}^{3\pi/4} \frac{(\pi-x)}{1+\sin(\pi-x)} dx.$$

$$\Rightarrow I = \int_{\pi/4}^{3\pi/4} \frac{(\pi-x)}{1+sinx} dx \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

❖ $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$\therefore a+b = \frac{\pi}{4} + \frac{3\pi}{4}$

(i) + (ii) হতে পাই,

$$2I = \int_{\pi/4}^{3\pi/4} \frac{(x+\pi-x)}{(1+sinx)} dx.$$

$$\Rightarrow I = \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1+\sin x} dx$$

$$= \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1-sinx}{\cos^2 x} dx.$$

$$= \frac{\pi}{2} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sec^2 x - secx.tanx)\, dx.$$

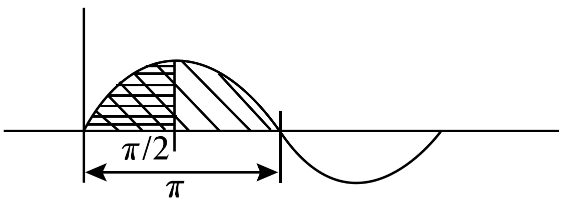
$$= \frac{\pi}{2} [tanx - secx]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$Type - 02$

$\int(a-x) = f(x)$ হলে, $\int_0^a f(x)dx = 2\int_0^{\frac{a}{2}} f(x).dx.$

$\int(a-x) = -f(x)$ হলে, $\int_0^a f(x)dx = 0$

❖ ***sin x* এর লেখ:**



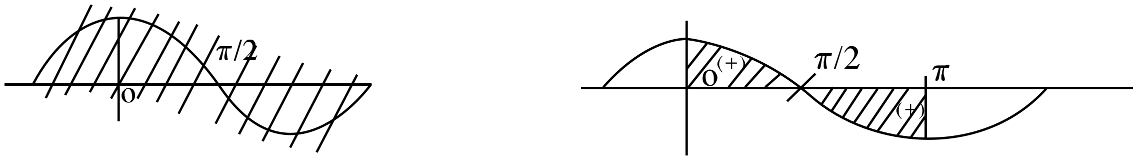
$f(x) = sinx$

$f(\pi - x) = sin(\pi - x) = sin\,x$

$\therefore \int_0^{\pi} sinx\, dx = 2 \qquad \int_0^{\frac{\pi}{2}} sinx\, dx$

$\int_0^a f(x)dx = 2\int_0^{\frac{a}{2}} f(x)dx$

❖ ***cos x*- এর লেখ:**



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$f(x) = cosx$
$f(\pi - x) = \cos(\pi - x) \quad = -cosx.$
$\therefore f(\pi - x) = -f(x)$
$\therefore \int_0^{\pi} cosx\, dx = 0;$

❖ $I = \int_o^{\pi} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x.}$

$f(x) = a^2 \sin^2 x + b^2 \cos^2 x$
$f(\pi - x) = a^2 \sin^2(\pi - x) + b^2 \cos^2(\pi - x)$
$= a^2 \sin^2 x + b^2 \cos^2 x$
$\therefore f(\pi - x) = f(x).$
$\therefore I = 2.\int_o^{\frac{\pi}{2}} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}$
$= 2\int_o^{\frac{\pi}{2}} \frac{\sec^2 x}{b^2 + a^2 \tan^2 x} dx$
$= \frac{2}{a^2} \int_o^{\infty} \frac{dz}{\left(\frac{b}{a}\right)^2 + z^2}$
$= \frac{2}{a^2} \left[\tan^{-1} \frac{az}{b}\right]_0^{\infty} . \frac{a}{b}$
$= \frac{2}{a^2} \times \frac{\pi}{2} . \frac{a}{b} = \frac{\pi}{ab}$

$z = \tan x$
$dz = \sec^2 x\, dx$

x	0	$\pi/2$
z	0	∞

❖ $I = \int_o^{\pi} \frac{x}{a^2 \sin^2 x + b^2 \cos^2 x} dx \ldots\ldots\ldots\ldots (i)$

প্রথমে লবকে x মুক্ত করা

$I = \int_o^{\pi} \frac{\pi - x}{a^2 \sin^2 x + b^2 \cos^2 x} dx \ldots\ldots\ldots\ldots (ii)$

(i) + (ii),

$2\text{I} = \pi \int_o^{\pi} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$
$\Longrightarrow I = \frac{\pi}{2} \int_o^{\pi} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx.$
$\Rightarrow I = \frac{\pi}{2} . \frac{\pi}{ab}$
$= \frac{\pi^2}{2ab}$

❖ $e^x = 1 + \frac{x}{\llcorner 1} + \frac{x^2}{\llcorner 2} + \frac{x^3}{\llcorner 3} + \frac{x^4}{\llcorner 4} + \cdots\cdots\cdots \infty$
$= \sum_{k=0}^{\propto} \frac{x^k}{\llcorner k}$

❖ $\int e^x dx = \int \sum_{k=0}^{\infty} \frac{x^k}{k!} dx$
$= \sum_{k=0}^{\infty} \frac{1}{k!} \int x^k dx$
$= \sum_{k=0}^{\infty} \frac{1}{k!} \frac{x^{k+1}}{k+1} + c$
$= \sum_{k=0}^{\infty} \frac{x^{k+1}}{(k+1)!} + c$
$= \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\ldots + \infty + c$ [যেখানে c এর মান 1]

(A) $\sum_{k=0}^{\propto} \frac{(-1)^k x^{2k}}{2^{2k}.(k!)^2}$ (B) $\sum_{k=0}^{\propto} \frac{(-1)^k}{(2k+1)k!}$ (C) $\sum_{k=0}^{\propto} \frac{(-1)^k.lnx}{\sqrt{k}.k!}$

(D) $\sum_{k=0}^{\propto} \frac{(-1)^{k+1}.x^k}{k!}$ (E) $\sum_{k=0}^{\propto} \frac{(-1)^k x^{-\frac{1}{2}}}{2lnk}$

❖ $\boldsymbol{I = \int_0^1 e^{-x^2} dx}$

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Solve: $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots\cdots\infty$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\therefore e^{-x^2} = \sum_{k=0}^{\infty} \frac{(-x^2)^k}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k . x^{2k}}{k!}$$

$$I = \int_0^1 e^{-x^2} dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{\lfloor k} \int_0^1 x^{2k} dx$$

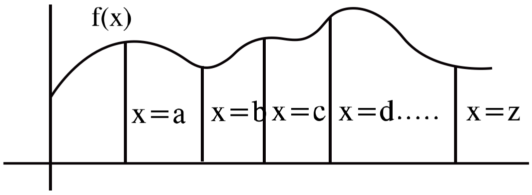
$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left[\frac{x^{2k+1}}{2k+1}\right]_0^1$$

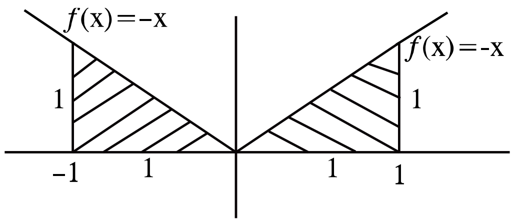
$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\frac{1}{2k+1} - 0\right)$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)k!}$$

Integration of Modulus function:

❖ $\int_a^z f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx + \int_c^d f(x)dx + \cdots \ldots\ldots\ldots\ldots + \int_y^z f(x)dx.$





Problem: $\int_{-1}^{1} |x| dx$

$|x| = +x;\ x \geq 0$

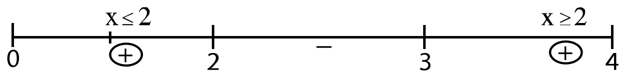
$|x| = -x;\ x < 0$

$= \int_{-1}^{0} -x dx + \int_0^1 x dx. \quad = -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1$

$= -\left[o - \frac{1}{2}\right] + \left[\frac{1}{2} - 0\right]$ $\therefore \int_{-1}^{1} |x| dx = 2 \times \left(\frac{1}{2} \times 1 \times 1\right) = 1$

$= \frac{1}{2} + \frac{1}{2} = 1.$

❖ **Problem**: $I = \int_0^4 |x^2 - 5x + 6|\, dx$

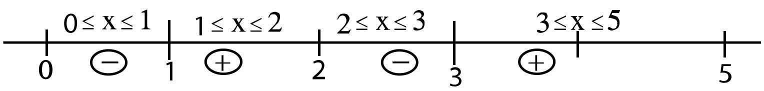


$= \int_0^4 |(x-2)(x-3)|\, dx$

$= \int_0^2 (x^2 - 5x + 6) dx + \int_2^3 -(x^2 - 5x + 6) dx. + \int_3^4 (x^2 - 5x + 6)\, dx.$

$= \left[\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right]_0^2 - \left[\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right]_2^3 + \left[\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right]_3^4$

❖ **Problem**: $I = \int_0^5 |(x-1)(x-2)(x-3)| dx$



$= -\int_0^1 (x^3 - 6x^2 + 11x - 6) dx + \int_1^2 (x^3 - 6x^2 + 11x - 6) dx - \int_2^3 (x^3 - 6x^2 + 11x - 6) dx - \int_3^5 (x^3 - 6x^2 + 11x - 6) dx$

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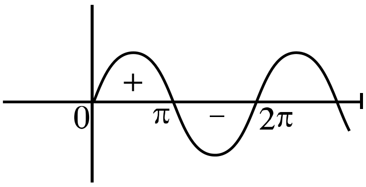
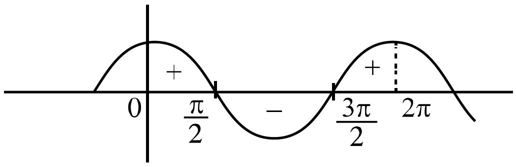
❖ **Problem:**

$$I = \int_0^{2\pi} |\sin x|\, dx$$
$$= \int_0^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} (-\sin x)\, dx$$
$$= [-\cos x]_0^{\pi} - [-\cos x]_{\pi}^{2\pi}$$



Problem: $I = \int_0^{2\pi} |\cos x|\, dx$

$$= \int_0^{\frac{\pi}{2}} \cos x\, dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\cos x\, dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x\, dx$$



$$= [sinx]_0^{\frac{\pi}{2}} - [sinx]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + [sinx]_{\frac{3\pi}{2}}^{2\pi}$$

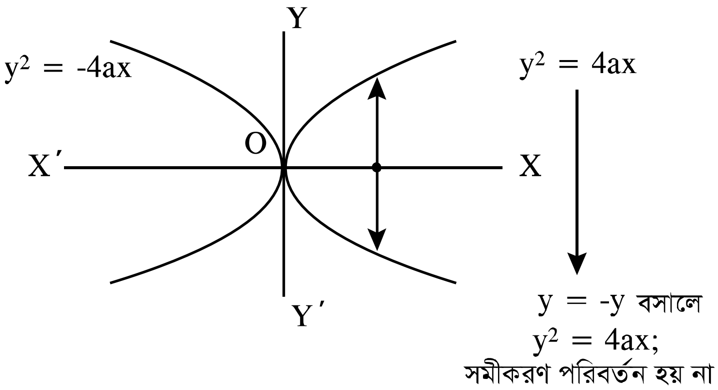
Proceed yourself

Area under curve

Area নির্ণয় করার আগে আমরা curve sketching শিখি:

Curve sketching:

Case-01: ***Symmetry about X axis.***



কোন একটি গ্রাফ X-axis এর সাপেক্ষে তখনই প্রতিসম হবে যখন y এর স্থলে –y বসালে সমীকরণটি অপরিবর্তিত থাকে।

Case-02: **Symmetry about Y axis.**

$x^2 = 4ay$

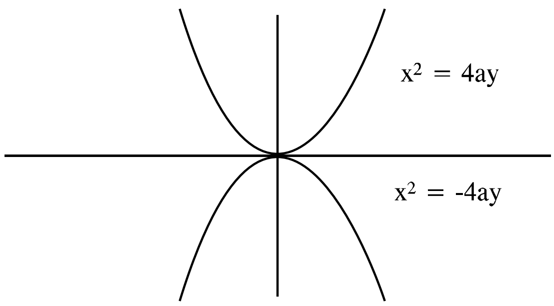
$x = -x$ বসালে,

$x^2 = 4ay$

অর্থাৎ সমীকরণটি পরিবর্তন হয় না

কোন একটি গ্রাফ Y-axis এর সাপেক্ষে তখনই Symmetry হবে

যখন সমীকরণে $x = -x$ বসালে সমীকরণের কোন পরিবর্তন হয় না।



Case-03: ***Symmetry about y = x line.***

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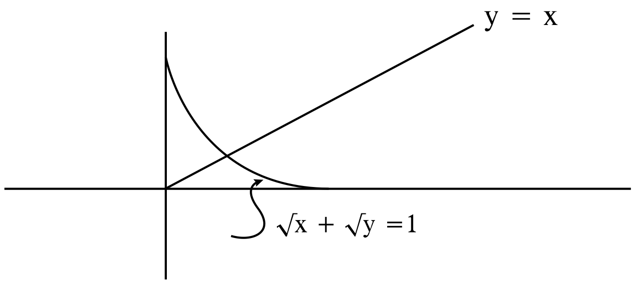
কোন গ্রাফ $y = x$ লাইনের সাপেক্ষে প্রতিসম হবে যদি সমীকরণে y এর স্থলে x এবং x এর স্থলে y বসাই, তবে সমীকরণের কোন পরিবর্তন হয় না।



x এর স্থলে y এবং y এর স্থলে x বসালে $\sqrt{y} + \sqrt{x} = 1$ অর্থাৎ সমীকরণের পরিবর্তন হয়নি।

Case-04: ***Symmetry about*** $\boldsymbol{y = -x}$ ***line***

কোন গ্রাফ $y = -x$ লাইনের সাপেক্ষে প্রতিসম হবে যদি x এর স্থলে $-y$ এবং y এর স্থলে $-x$ বসালে সমীকরণের কোন পরিবর্তন হয় না।

$x^2 + y^2 - 2x + 2y + 1 = 0$ বৃত্ত, কেন্দ্র (1,-1)

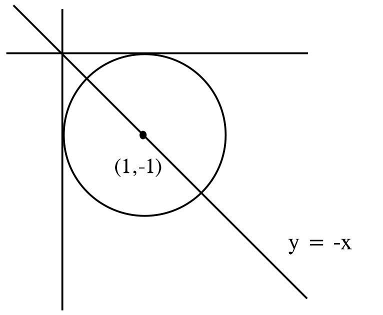
x এর স্থলে –y

y এর স্থলে –x বসায়

$y^2 + x^2 + 2y - 2x + 1 = 0$

$\therefore x^2 + y^2 - 2x + 2y + 1 = 0$

অর্থাৎ সমীকরণের কোন পরিবর্তন হয় না।



Case-05: **Symmetry about origin**:

$y = x^3$

কোন গ্রাফ origin লাইনের সাপেক্ষে প্রতিসম হবে যদি x এর স্থলে $-x$ এবং y এর স্থলে $-y$ বসালে সমীকরণের কোন পরিবর্তন হয় না।

$y = x^3$

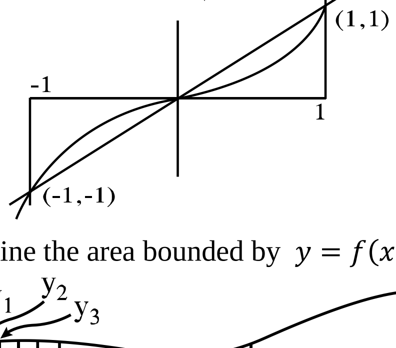
$x \to -x$

$y \to -y$

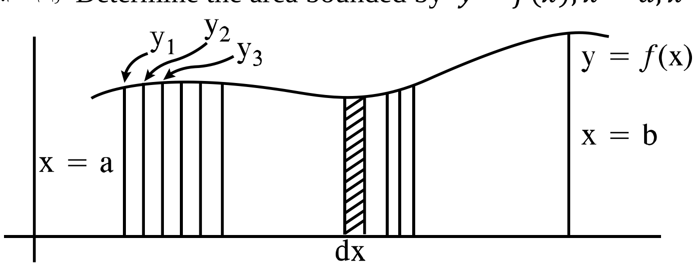
$-y = -x^3$

$\Rightarrow y = x^3$

অর্থাৎ সমীকরণটির কোন পরিবর্তন হয়নি



General আলোচনা: area নির্ণয়ের জন্য Determine the area bounded by $y = f(x); x = a; x = b;$ and $x\text{-}axis$



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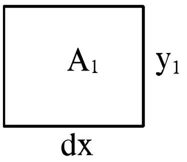
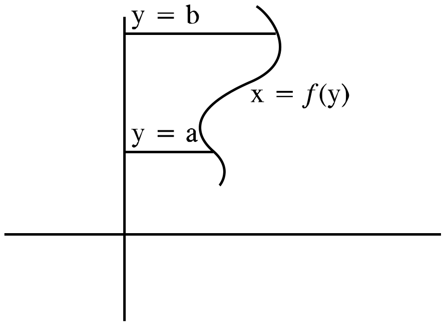
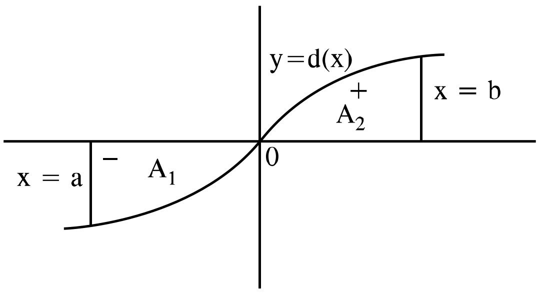


$A_1 = y_1 . dx$
$A_2 = y_2 . dx$
$A_3 = y_3 dx$
$A = A_1 + A_2 + \cdots\cdots\cdots$
$= (y_1 + y_2 + y_3 + \cdots \ldots \ldots \ldots .) dx.$
$\therefore A = \sum_{x=a}^{a=b} y \, dx$
$A = \int_a^b y . dx$ means an area bounded by $y = f(x), x = a, x = b$ and x axis.



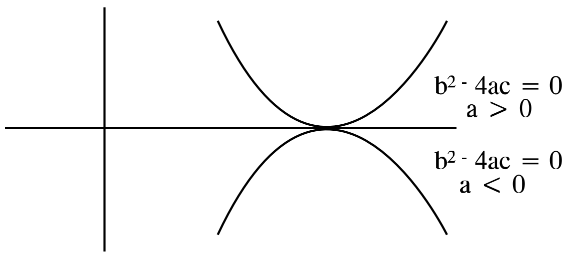
$$A = \int_a^b x \, dy.$$

2.



$A_1 = \int_a^0 -f(x) dx$
$A_2 = \int_0^b f(x) dx$
$A = \left| \int_a^0 f(x) dx \right| + \int_0^a f(x) dx$

❖ **Problem**: Find the area by the curve y=x(x-3) and x-axis
$f(x) = ax^2 + bx + c$
Solve:
★ Parabolic function
★ $b^2 - 4ac = 0$
> 0
< 0



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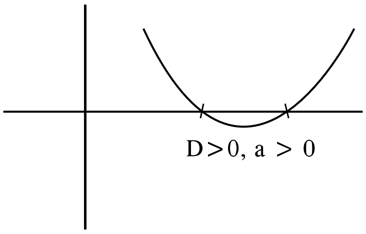
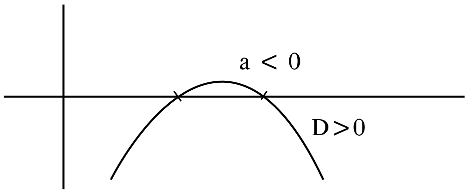
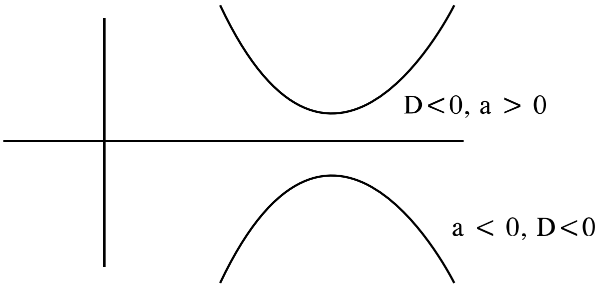
★ $\boldsymbol{b^2 - 4ac > 0}$





★ $\boldsymbol{b^2 - 4ac < 0}$

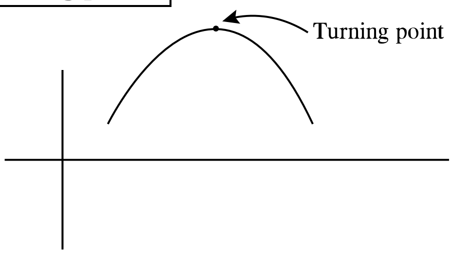


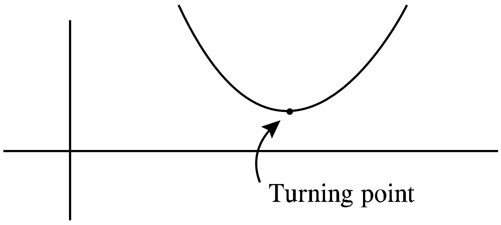
★ $\boldsymbol{y = ax^2 + bx + c}$

$y = 0$ হলে, x এর যে মান গুলো পাওয়া যায়, সেই মানগুলোই x অক্ষের ছেদ বিন্দু।

$x = 0$ হলে, y এর যে মান পাওয়া যায়, x অক্ষের সেই বিন্দুতে ছেদ করবে।

Turning point:





কোন একটা function এর turning point বলতে বুঝায়, যে point এ ঢাল হয় শূণ্য।

অর্থাৎ $\frac{dy}{dx} = 0$ হয় যে সব বিন্দুতে সেই সব বিন্দুই turning point.

❖ **Problem**: $y = x(x-3)$

$y = 0$ হলে $x = 0,\ 3$

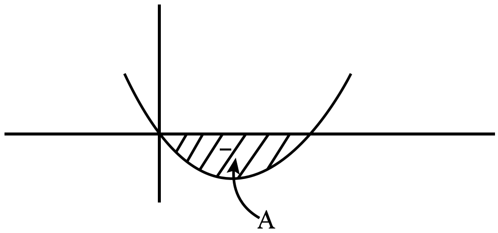
$x = 0$ হলে $y = 0.$

$y = x^2 - 3x$

$b^2 - 4ac = (-3)^2 - 4 \times 1 \times 0 = 9 > 0$

$\therefore b^2 - 4ac > 0;$

x^2 -এর সহগ > 0; upard



$$A = \left|\int_0^3 y dx\right|$$

$$= \left|\int_0^3 (x^2 - 3x)\right| dx$$

$$= \left|\left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_0^3\right|$$

$$= \left|\frac{27}{3} - \frac{3\times 9}{2}\right|$$

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$= |9 - 13.5| = 4.5$ বর্গ একক

❖ **Problem:** Find the area by the curve $y = x(4 - x)$ and x-axis from $x = 0\ to\ x = 5$

$y = 0$ হলে, $x = 0,4$

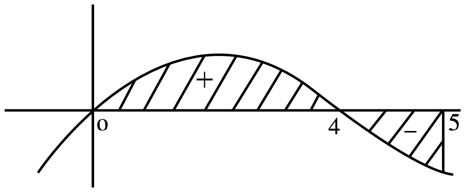
$x = 0$ হলে, $y = 0$

$y = 4x - x^2 = -x^2 + 4x$

$D = 16 - 4(-1)0 = 16 > 0$

$b^2 - 4ac = 16 - 0 = 16 > 0$

x^2 এর সহগ negative, downward.



$A = \int_0^4 y\,dx + |\int_4^5 y\,dx|$

$= \int_0^4 (4x - x^2)dx + \left|\int_4^5 (4x - x^2)dx\right|$

$= \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 + |\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_4^5|$

$= \left(32 - \frac{64}{3}\right) + \left|\left(50 - \frac{125}{3}\right) - \left(32 - \frac{64}{3}\right)\right|$

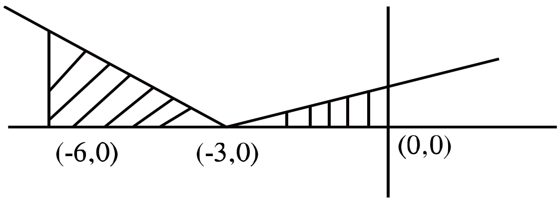
$= \frac{32}{3} + \left|\frac{25}{3} - \frac{32}{3}\right|$

$= \frac{32}{3} + \frac{7}{3}$

$= \frac{39}{3}$

$= 13$ বর্গ একক

❖ **Problem:** **Find the area by the graph $y = |x + 3|$ and x-axis from $x = -6$ to $x = 0$**



Solve:

$y = 0$ হলে, $x = -3$

$Area = \int_{-6}^{-3} y\,dx + \int_{-3}^{0} y\,dx$

$|x + 3| = (x + 3)\ ; x \geq -3$

$[x + 3] = -(x + 3); x < -3$

$= \int_{-6}^{-3} -(x + 3)dx + \int_{-3}^{0} (x + 3)dx$.

$= -\left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$

$= -\left\{\left(\frac{9}{2} - 9\right) - (18 - 18)\right\} + \left\{0 - \left(\frac{9}{2} - 9\right)\right\}$

$= \frac{9}{2} + \frac{9}{2}$

$= 9\ sq.unit$

❖ **Problem:** Find the area by the graph $y = |x + 1| + 1 and$ x-axis from $x = -3\ to\ x = 3$

Solve:

$y = 0\ ;\ |x + 1| = -1$

$\Rightarrow \pm(x + 1) = -1$

$\Rightarrow x + 1 = \pm 1$

$\Rightarrow x = \pm 1 - 1$

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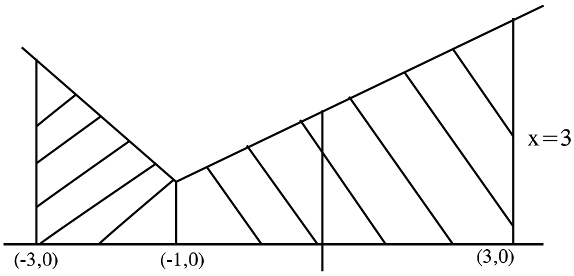


$Area = \int_{-3}^{-1} y\,dx + \int_{-1}^{3} y\,dx.$

$= \int_{-3}^{-1}\{-(x+1)+1\}dx + \int_{-1}^{3}\{(x+1)+1\}dx$

$= -\int_{-3}^{-1} xdx + \int_{-1}^{3}(x+2)dx$

$= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2}+3x\right]_{-1}^{3}$

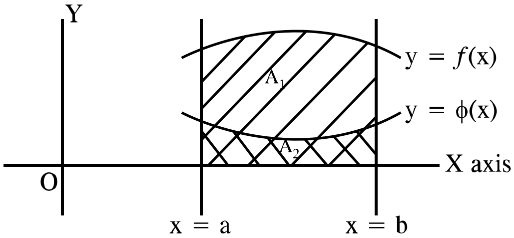
$= \square$

$|x+1| = x+1\,; x \geq -1$

$= -(x+1); x < -1.$

*বাকিটা পারা যাবে।

Case-1: কোন বিন্দুতে গ্রাফ ছেদ করে না।



যখন দুটি graph পরস্পরকে ছেদ করবে না তখন মনে রাখতে হবে definitely definitely limit দেয়া থাকবে।

$\int_{a}^{b} f(x)dx = A_1 + A_2.$

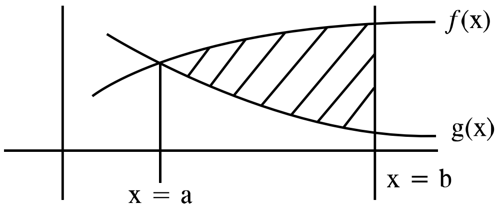
$\Longrightarrow \int_{a}^{b} f(x)dx = A_1 + \int_{a}^{b} \phi(x)dx.$

$\Longrightarrow A_1 = \int_{a}^{b} f(x)dx - \int_{a}^{b} \phi(x)dx \quad = \int_{a}^{b}\{f(x)-\phi(x)\}dx$

A_1 সর্বদা ধনাত্মক। দুইটি curve দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল হবে

তাই $A = \left|\int_{a}^{b}\{f(x)-\phi(x)\}dx\right|$

Case-2: গ্রাফদ্বয় একটা বিন্দুতে ছেদ করে। তাহলে অন্য limit দেয়া থাকবে।



$A = \left|\int_{a}^{b}[f(x)-g(x)]dx\right|$

❖ **Case-3:** গ্রাফদ্বয় দু'টি বিন্দুতে ছেদ করে। ছেদ বের করে x এর মান বের করব; এরা হবে লিমিট।

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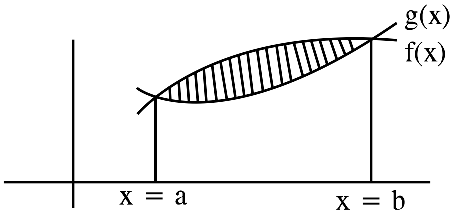
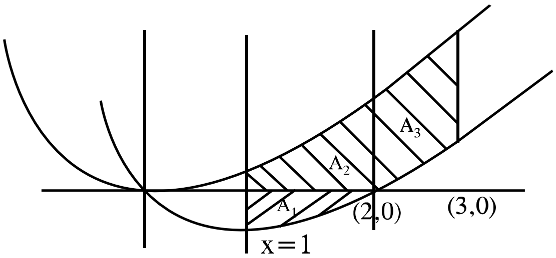


$$A = \left|\int_a^b \big(f(x) - g(x)\big)dx\right|$$

❖ **Problem:** **Find the area under the curve $y = x^2$; $y = x^2 - 2x$ and $x = 1$ to $x = 3$**

Solve:



$y = x^2$ এর $turning\ point; \frac{dy}{dx} = 2x = 0 \Rightarrow x = 0$

$y = x^2 - 2x$ এর $turning\ point; \frac{dy}{dx} = 2x - 2 = 0 \Rightarrow x = 1$

$A = A_1 + A_2 + A_3.$

$= \left|\int_1^2 (x^2 - 2x)dx\right| + \int_1^2 x^2.dx + \int_2^3 \{x^2 - (x^2 - 2x)\}dx$

$= \left|\left[\frac{x^3}{3} - \frac{2x^2}{2}\right]_1^2\right| + \left[\frac{x^3}{3}\right]_1^2 + [x^2]_2^3$

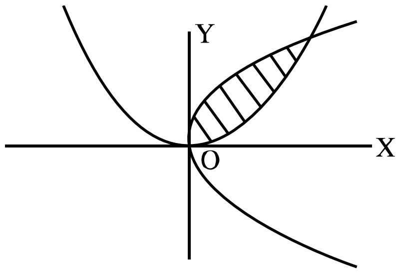
$= \left|\left(\frac{8}{3} - 4\right) - \left(\frac{1}{3} - 1\right)\right| + \left(\frac{8}{3} - \frac{1}{3}\right) + (9 - 4)$

$= \left|-\frac{4}{3} + \frac{2}{3}\right| + \frac{7}{3} + 5 = \frac{2}{3} + \frac{7}{3} + 5$

$= 8$ বর্গ একক।

❖ **Problem:** **Find the area under the curve, $y^2 = 4ax$ and $x^2 = 4ay$.**

Solve:



$y^2 = 4ax$ এবং $x^2 = 4ay$ সমাধান করে $x -$ এর মান বের করি।

$y^2 = 4ax$
$x^2 = 4ay$

$\left(\frac{x^2}{4a}\right)^2 = 4ax.$

$\Rightarrow x^4 = 64a^3x.$

$\Rightarrow x(x^3 - 64a^3) = 0$

$\therefore x = 0, 4a.$

$$A = \int_0^{4a} \left(2\sqrt{a}.\sqrt{x} - \frac{x^2}{4a}\right)dx$$

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$= 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{4a} - \left[\frac{x^3}{12a}\right]_0^{4a}$

$= \frac{4\sqrt{a}}{3} \times 8a^{\frac{3}{2}} - \frac{64a^3}{12a}$

$= \frac{32}{3}a^2 - \frac{16}{3}a^2$

$= \frac{16}{3}a^2$ বর্গ একক।

❖ **Problem:** **Find the area under the graph,** $\boldsymbol{y^2 = 4ax \ and\ y = mx}$.

Solve:

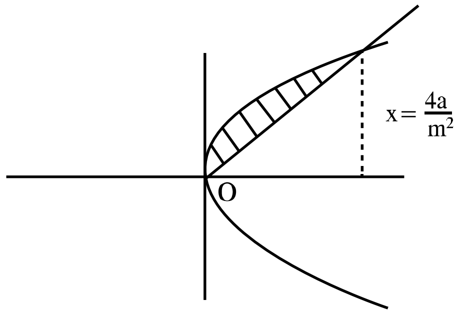
সমীকরণদ্বয় সমাধান করি।

$\left.\begin{aligned} y^2 &= 4ax \\ y &= mx \end{aligned}\right\}$ $(mx)^2 = 4ax.$

$\Rightarrow m^2x^2 - 4ax = 0$

$\Rightarrow x(m^2x - 4a) = 0$

$\therefore x = 0, \frac{4a}{m^2}$



$Area = \int_0^{\frac{4a}{m^2}} \left(2\sqrt{a}.\sqrt{x} - mx\right)dx.$

$= \left[2\sqrt{a}\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{mx^2}{2}\right]_0^{\frac{4a}{m^2}}$

$= \frac{4}{3}\sqrt{a}.8\frac{a^{\frac{3}{2}}}{m^3} - \frac{m}{2} \times \frac{16a^2}{m^4}$

$= \frac{32}{3}\frac{a^2}{m^3} - 8\frac{a^2}{m^3}$

$= \frac{8}{3} \times \frac{a^2}{m^3}$

$y^2 = 4ax$ এবং $y = mx$ দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $\frac{8a^2}{3m^3}$

❖ **Problem:** $\boldsymbol{y^2 = 16x}$ এবং $\boldsymbol{y = 3x}$ **দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল কত?**

$a = 4;\ \ m = 3$

$A = \frac{8}{3} \times \frac{a^2}{m^3}$

$= \frac{8\times16}{3\times3^3}$

$= \frac{128}{81}$

Homework: $\boldsymbol{x^2 = 4ay}$ *এবং* $\boldsymbol{y = mx}$ **দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল কত?**

✍ **<u>D.U হলে ও অন্যান্য University এর জন্য গুরুত্বপূর্ণঃ</u>**

✍ $\int_2^5 f(x)dx = 5\ \ \int_1^4 f(x+1)dx = ?$

Ans: 5

graph এর মাধ্যমে ব্যাখ্যাঃ $\int_b^a f(x)dx = \int_{b-c}^{a-c} f(x+c)dx$

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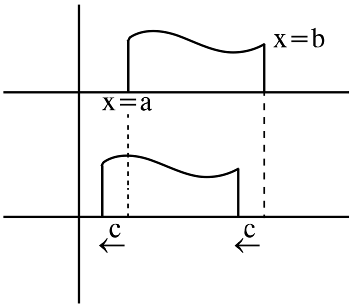
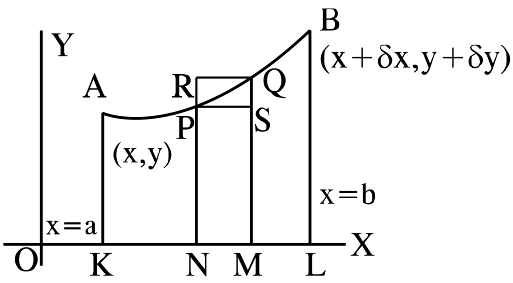


সম্পূর্ণ ক্ষেত্রফল অংশটি c একক পরিমাণ কমে সরে যাচ্ছে। [graph shifting concept]

আবর্তিত ঘনবস্তুর আয়তন:



মনে করি, $y = f(x)$ সমীকরণ দ্বারা প্রদত্ত বক্ররেখা AB এবং $x = a$ ও $x = b$ দ্বারা বর্ণিত রেখা দুইটি যথাক্রমে $P(x, y)$ এবং $Q(x + \delta x, y + \delta y)$ $PN \perp OX$, $QM \perp OX$, $SR \perp RN$, $PS \perp QM$

মনে করি, AKNPA এবং AKMQPA ক্ষেত্র দুটি X অক্ষের চর্তুদিকে ঘূর্ণনের ফলে সৃষ্ট ঘনফল δV। এখন, $PN = y$, $QM = y + \delta y, NM = \delta x$, অতএব, RNMQ ক্ষেত্রটি ঘূর্ণনের ফলে সৃষ্ট ঘনফল $= \pi(y + \delta y)^2\delta x$

চিত্রে, RNMQ ক্ষেত্রদ্বারা সৃষ্ট আয়তন > PNMQ ক্ষেত্রদ্বারা সৃষ্ট আয়তন > PNMS দ্বারা সৃষ্ট আয়তন

অর্থাৎ,

$\pi\ (y + \delta y)^2\ \delta x > \delta v > \pi y^2\delta x$

$\Rightarrow \pi\ (y + \delta y)^2 > \frac{\delta v}{\delta x} > \pi y^2$

$Q \rightarrow P$ হলে $\delta x \rightarrow o$

$\frac{dv}{dx} = \pi y^2 \Rightarrow dv = \pi y^2 dx$

$\therefore \int_a^b dv = \int_a^b \pi y^2 dx$

$\therefore y = f(x)$, x অক্ষ, $x = a$ ও $x = b$ দ্বারা আবদ্ধ ক্ষেত্রটি x অক্ষের চর্তুদিকে ঘূর্ণনের ফলে সৃষ্ট ঘনফল $\int_a^b \pi y^2 dx$ হয়।

অনুরূপ ভাবে Y অক্ষের ক্ষেত্র হবে $\int_a^b \pi x^2 dy$

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